

2<sup>nd</sup> Workshop on Frontier Modeling of Energy & Environment (FMEE)

# Directional Marginal Abatement Cost of Air Pollutants in the Coal-Fired Power Industry

(燃煤電廠污染排放物之方向性邊際減排成本)

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- Background
- Literatures
- Directional Marginal Product
- Marginal Product with Bad Output
- Marginal Abatement Costs of Pollutants
- Empirical Study
- Conclusions

# Background

- CO2 Emission

- Total U.S. energy-related emissions of carbon dioxide (CO<sub>2</sub>) by the electric power sector in 2012 were 2,039 million metric tons, or about 77% of total U.S. CO<sub>2</sub> emissions.
- CO2 emissions from U.S. electricity generation by source, 2012

Source	Million Metric Tons	Share of Total
Coal	1,514	74%
Natural gas	494	24%
Petroleum	19	1%
Other <sup>2</sup>	12	1%
Total	2,039	

<sup>2</sup>Miscellaneous wastes and from geothermal power generation.

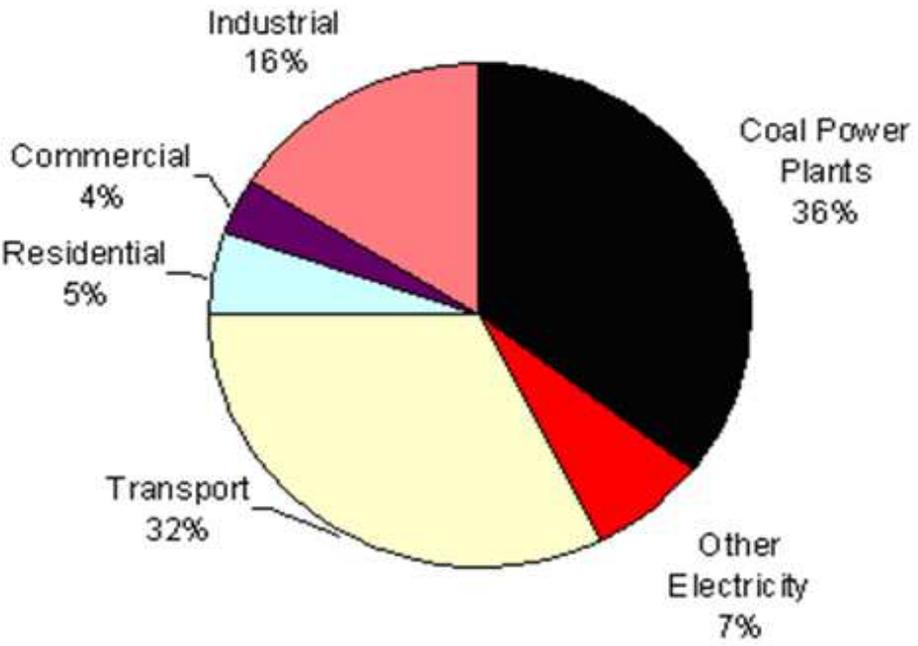
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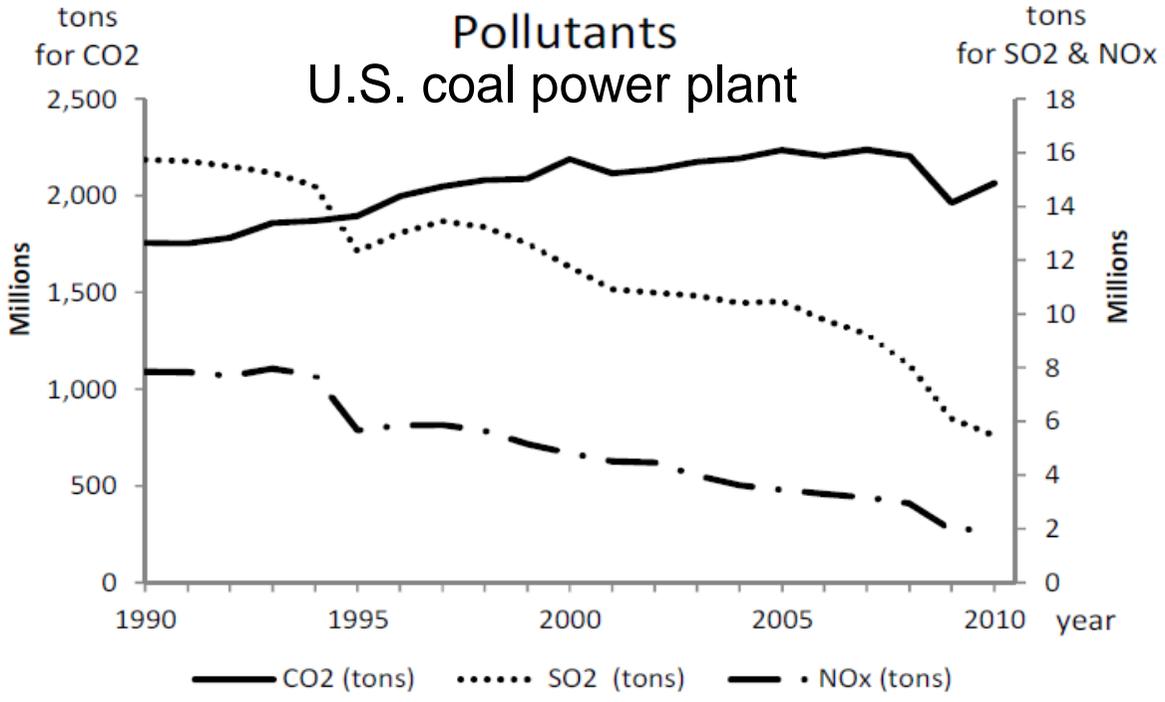
# Source of Air Pollution

- U.S. Coal-fired Power Plant

U.S. CO2 Emissions by Sector and Source, 2030  
(EIA AEO 2009 Projections)



77% (2012) → 43% (2030)



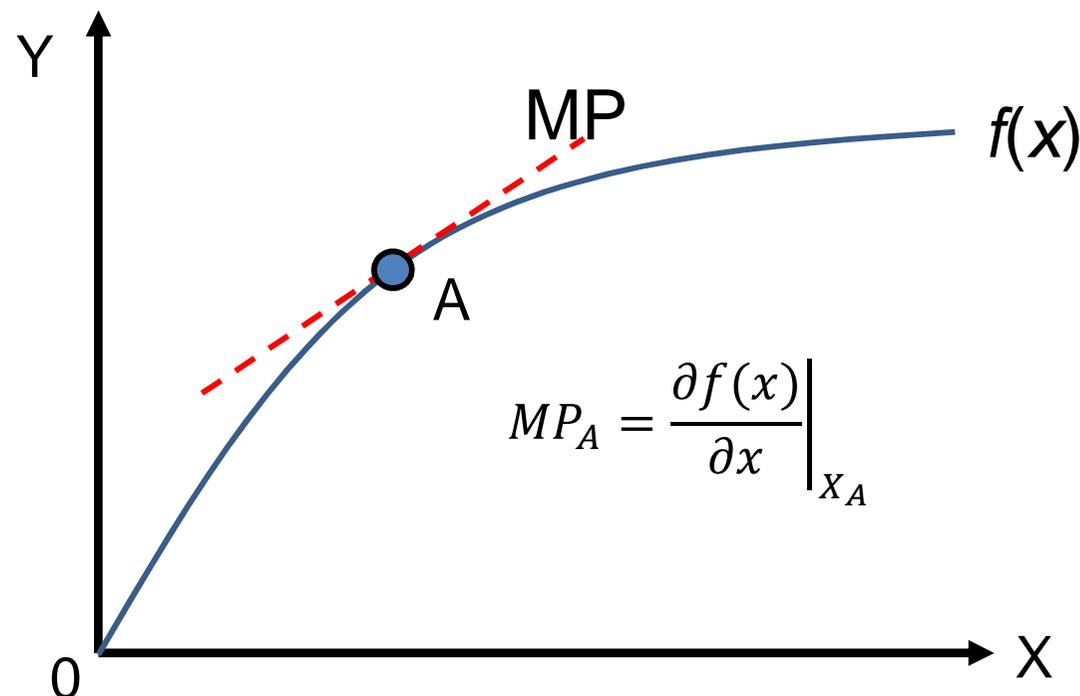
# Cap-and-Trade Regulation

- Economic solutions to environmental **externalities**, such as the air pollution, often include emissions taxes and permit trading systems.
- Policy-makers need to determine the **marginal abatement costs** (MAC) or **shadow prices** (SP) of pollutants to represent the costs of reducing one extra unit of pollutant.
- The emission trading mechanism is based on Coase's assertion (Coase, 1960) that if trading in an externality and absent a transaction cost, **bargaining** will lead to an efficient outcome regardless of the initial allocation of property rights trading.

How much does it cost to abate  
one ton of pollutant emission?

# Literatures

- The shadow prices (SP) of pollutants are used as a **reference value** to the allowance price in the emission trading market (Lee et al., 2002).
- We can estimate the **marginal product (MP)** to derive the shadow price (Keilback, 1995), which is a differentiable characteristic of the production function.



- Profit Maximization

$$\pi(p_y, p_b, p_x) = \max_{y,b,x} p'_y y - p'_b b - p'_x x$$

$$\text{s. t. } F(x, y, b) = 0 \quad (\text{Production Transformation Function})$$

- Lagrange function:  $\max_{y,b,x} p'_y y - p'_b b - p'_x x + \varphi F(x, y, b)$
- First-order conditions (FOCs):

$$\triangleright p_{y_j} + \varphi \frac{\partial F(x,y,b)}{\partial y_j} = 0$$

$$\triangleright -p_{b_q} + \varphi \frac{\partial F(x,y,b)}{\partial b_q} = 0$$

$$\triangleright -p_{x_i} + \varphi \frac{\partial F(x,y,b)}{\partial x_i} = 0$$

$$\triangleright F(x, y, b) = 0$$

- Marginal Abatement Costs

- $$p_{b_q} = p_{y_j} \left( \frac{\partial F(x,y,b)}{\partial b_q} / \frac{\partial F(x,y,b)}{\partial y_j} \right)$$

- $$p_{b_q} = p_{y_j} \left( \frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial b_q} / \frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial y_j} \right)$$

- The equation is used to estimate the shadow price  $p_b$  of pollutant  $b$ , where  $p_y$  is the price of desirable output  $y$  and  $\vec{D}_O(x,y,b;g^y,g^b)$  is the directional output distance function (see Lee et al., 2002; Färe et al., 2005).

- How to estimate  $\frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial b_q}$  or  $\frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial y_j}$ ?

- **Stochastic Frontier Analysis (SFA) (Färe et al., 2005)**

- Parametric method
- Translog functional form
- Directional distance function

$$\begin{aligned} \ln D(x, y, b) &= \alpha_0 + \sum_i \alpha_i \ln x_i + \sum_j \alpha_j \ln y_j + \sum_k \alpha_k \ln b_k \\ &+ \frac{1}{2} \sum_i \sum_{i'} \gamma_{ii'} \ln x_i \ln x_{i'} + \frac{1}{2} \sum_j \sum_{j'} \gamma_{jj'} \ln y_j \ln y_{j'} \\ &+ \frac{1}{2} \sum_k \sum_{k'} \gamma_{kk'} \ln b_k \ln b_{k'} + \sum_j \sum_k \gamma_{jk} \ln y_j \ln b_k \\ &+ \sum_i \sum_j \beta_{ij} \ln x_i \ln y_j + \sum_i \sum_k \beta_{ik} \ln x_i \ln b_k \gamma_{ii'} \\ &= \gamma_{ii'}, i \neq i'; \gamma_{jj'} = \gamma_{j'j}, j \neq j'; \gamma_{kk'} = \gamma_{k'k}, k \neq k' \end{aligned}$$

$$\begin{aligned} \text{Min } & \sum_n \left[ \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) - 0 \right] \\ \text{s.t. } & \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) \geq 0; \\ & \partial \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) / \partial y^n \leq 0; \\ & \partial \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) / \partial b^n \geq 0; \\ & \partial \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) / \partial x^n \geq 0; \\ & g_y \sum_j \alpha_j - g_b \sum_k \alpha_k = -1; \\ & g_y \sum_j \sum_{j'} \gamma_{jj'} - g_b \sum_j \sum_k \gamma_{jk} = 0; \\ & g_y \sum_j \sum_k \gamma_{jk} - g_b \sum_k \sum_{k'} \gamma_{kk'} = 0; \\ & g_y \sum_i \sum_j \beta_{ij} - g_b \sum_i \sum_k \beta_{ik} = 0; \\ & g_y^2 \sum_j \sum_{j'} \gamma_{jj'} + g_b^2 \sum_k \sum_{k'} \gamma_{kk'} - g_y g_b \sum_j \sum_k \gamma_{jk} = 0; \\ & \gamma_{ii'} = \gamma_{i'i}, i \neq i'; \gamma_{jj'} = \gamma_{j'j}, j \neq j'; \gamma_{kk'} = \gamma_{k'k}, k \neq k' \end{aligned}$$

- **Data Envelopment Analysis** (Lee et al. 2002)
  - Nonparametric method
  - Directional distance function
  - Dual variables

$$\begin{aligned}\vec{D}_o(x, y, b; g_y, g_b) &= \max_{\lambda, \beta} \beta \\ \text{s.t. } Y\lambda &\geq (1 + \beta g_y)y^n; \\ B\lambda &= (1 - \beta g_b)b^n; \\ X\lambda &\leq x^n; \\ \beta, \lambda &\geq 0\end{aligned}$$

However...

Previous studies have estimated the shadow prices of individual undesirable outputs **separately**.

- This equation  $p_b = p_y \left( \frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial b} / \frac{\partial \vec{D}_O(x,y,b;g^y,g^b)}{\partial y} \right)$ , which takes derivatives with respect to one specific undesirable output to estimate its shadow price, implicitly assumes that a firm can generate only one type of pollutant at a time when increasing one extra unit of input.
- That is, estimating the shadow price of SO<sub>2</sub> is independent of estimating the shadow price of NO<sub>x</sub>.
- In reality, the production process generates multiple undesirable outputs simultaneously when producing desirable outputs. Thus, estimating shadow prices separately may lead to an overestimation of marginal productivity and an underestimation of shadow price.



How to estimate the **marginal effects** of multiple good and bad outputs when increasing one extra unit of input?

# Marginal Product (MP)

- Marginal Product  $MP_A = \left. \frac{\partial f(x)}{\partial x} \right|_{X_A}$
- Podinovski and Førsund (2010) propose a directional derivative technique to assess the marginal product of a **nondifferential efficient frontier** constructed by the data envelopment analysis (DEA) estimator.

$$\frac{\partial Y_{j^*r}}{\partial X_{i^*r}} = \beta_{i^*j^*r}^{+DEA} = \text{Min } v_{i^*}$$

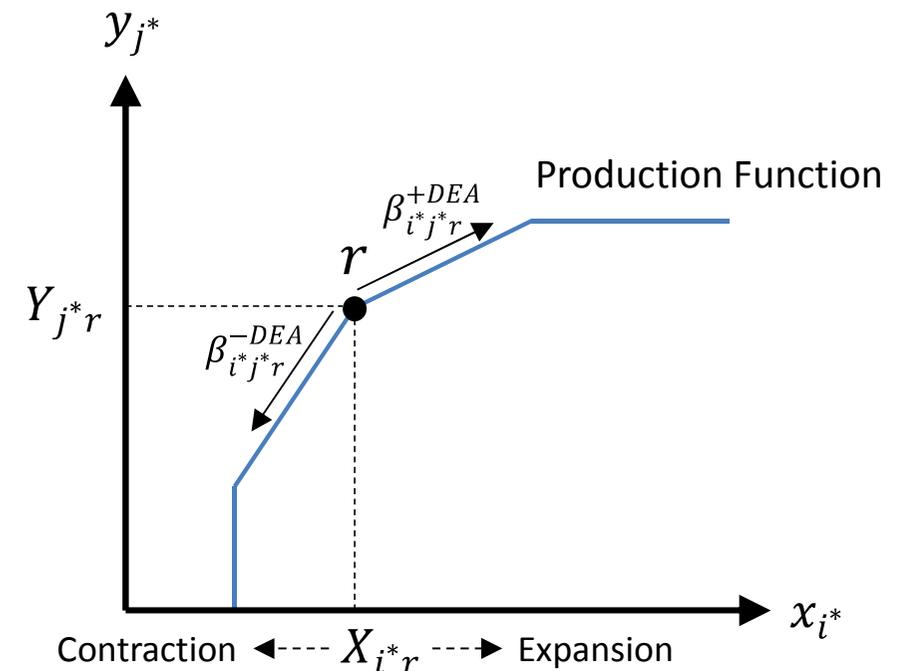
Subject to

$$\sum_i v_i X_{ir} - \sum_j u_j Y_{jr} + u_0 = 0$$

$$\sum_i v_i X_{ik} - \sum_j u_j Y_{jk} + u_0 \geq 0, \forall k$$

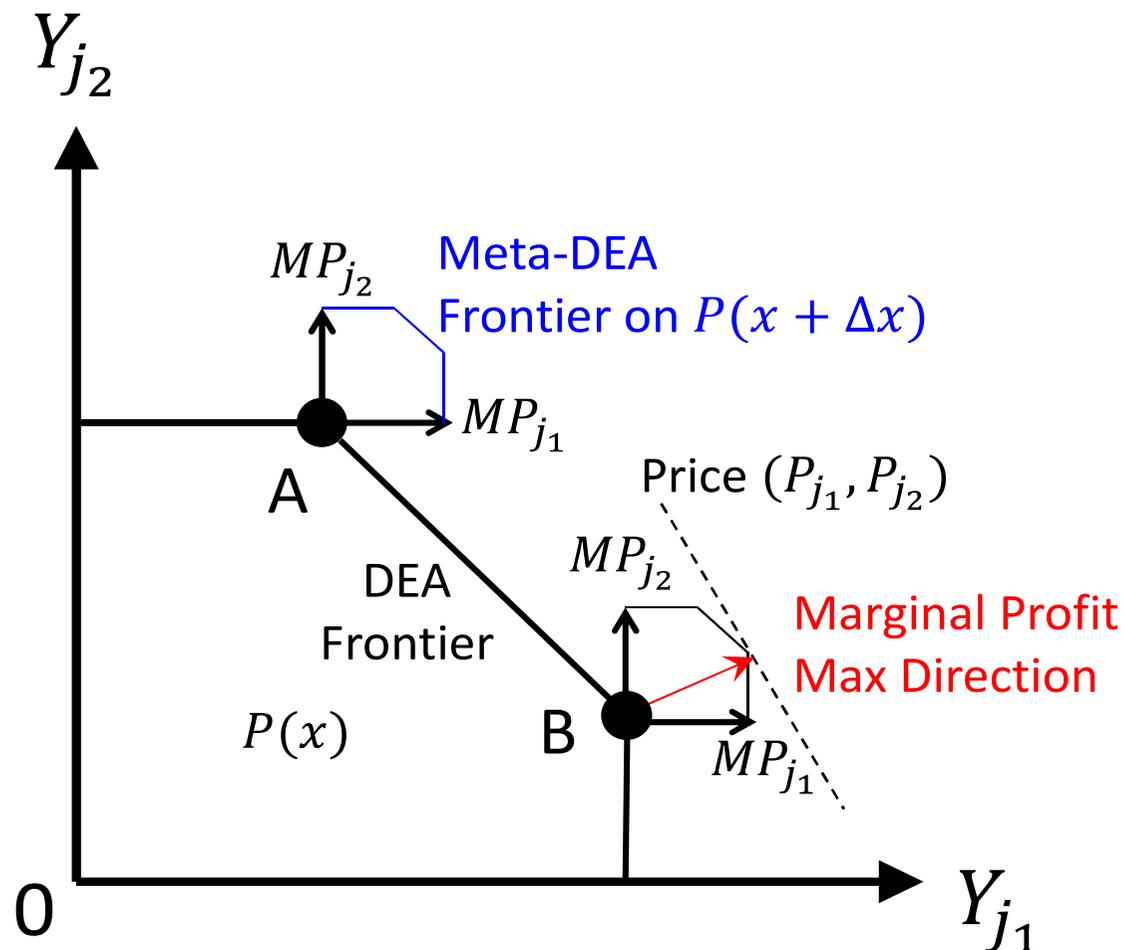
$$u_j^* = 1$$

$$v_i, u_j \geq 0, u_0 \text{ is free}$$



# Directional Marginal Product (DMP)

- Directional Marginal Product (DMP) (Lee, 2014)
  - Given  $(g^{y_1}, g^{y_2})$  a variant of directional distance function technique can generate this “direction”.



# Directional Marginal Product with Bad Output

- Kuosmanen and Podinovski (2009) introduce the **weak disposability** property which forms a convex technology with undesirable outputs.
- MP for multiple outputs given a pre-determined direction  $(g^{Y_j}, g^{B_q})$

Min  $v_{i^*}$

$$\text{s.t. } \sum_i v_i \frac{X_{ir}}{X_i^{Max}} - \sum_j u_j \frac{Y_{jr}}{Y_j^{Max}} + \sum_q w_q \frac{B_{qr}}{B_q^{Max}} + u_0 = 0$$

$$\sum_i v_i \frac{X_{ik}}{X_i^{Max}} - \sum_j u_j \frac{Y_{jk}}{Y_j^{Max}} + \sum_q w_q \frac{B_{qr}}{B_q^{Max}} + u_0 \geq 0, \forall k$$

$$\sum_i v_i \frac{X_{ik}}{X_i^{Max}} + u_0 \geq 0, \forall k$$

$$\sum_{j \in J^*} u_j g^{Y_j} + \sum_{q \in Q^*} w_q g^{B_q} = 1$$

$$v_i, u_j \geq 0, w_q, u_0 \text{ are free}$$

$$\frac{\partial(Y_{jr}, B_{qr})}{\partial X_{i^*r}} = v_{i^*} (g^{Y_j} Y_j^{Max}, -g^{B_q} B_q^{Max}) / X_{i^*}^{Max}$$

**Directional Marginal Product (DMP)**

Note:  $\sum_{j \in J^*} g^{Y_j} + \sum_{q \in Q^*} g^{B_q} = 1$  for unit simplex (Färe et al., 2013)



# Marginal Abatement Costs of Pollutants

- Shadow Prices of Pollutants  $p_{b_q}$
- $p_{b_q} = p_{y_j} \left( \frac{\partial F(x,y,b)}{\partial b_q} / \frac{\partial F(x,y,b)}{\partial y_j} \right) = p_{y_j} \left( \frac{\partial y_j}{\partial b_q} \right) = p_{y_j} \left( \frac{\partial y_j}{\partial x_i} / \frac{\partial b_q}{\partial x_i} \right)$
- Given direction vector  $(g^{Y_j}, g^{B_q})$
- $\left( \frac{\partial y_j}{\partial x_i}, \frac{\partial b_q}{\partial x_i} \right)$ : DMP of good output and bad output

$$\frac{\partial(Y_{jr}, B_{qr})}{\partial X_{i^*r}} = v_{i^*} (g^{Y_j} Y_j^{Max}, -g^{B_q} B_q^{Max}) / X_{i^*}^{Max}$$

Directional Marginal Product (DMP)



Directional Shadow Price (DSP)

# Empirical Study

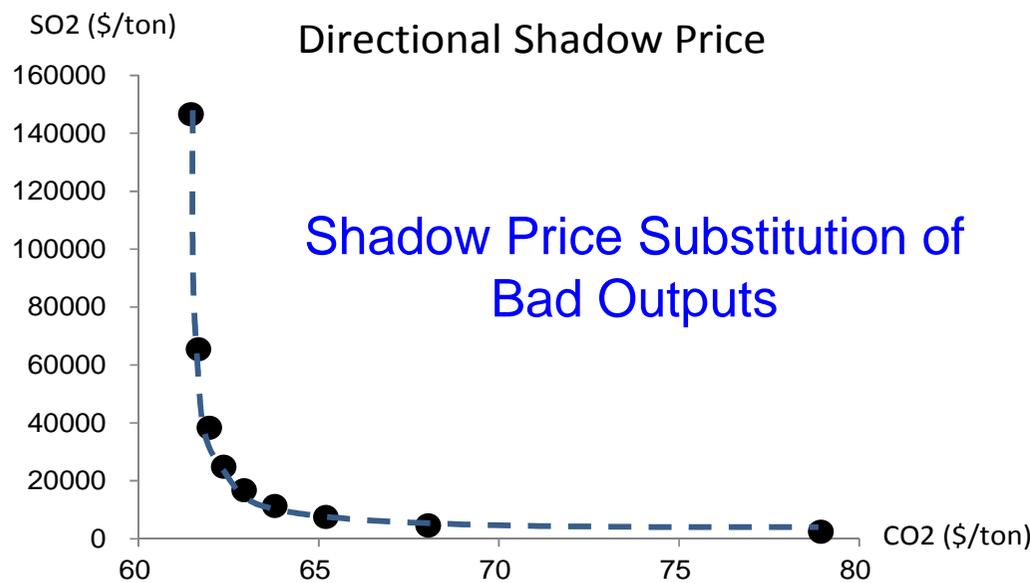
- 2010 US Coal Power Plants
- 48 observations of state-level dataset
- Inputs and Outputs
  - **One desirable output:** the annual amount of **electricity** generated by coal in Megawatt-hours (MWh).
  - **Three undesirable outputs:** the annual amount in tons of **CO<sub>2</sub>, SO<sub>2</sub> and NO<sub>x</sub>**.
  - **One input:** the annual amount in tons of **coal consumption**.
  - The average **electricity price ( $p_y$ )** is a weighted calculation among residential, commercial and industrial prices measured in dollars per MWh.

# Empirical Study

- DMP and DSP of CO<sub>2</sub> and SO<sub>2</sub>

Direction	DMP	DSP
$(g^{B_{CO_2}}, g^{B_{SO_2}}, g^{B_{NOx}})$	$\frac{\partial(B_{CO_2}, B_{SO_2}, B_{NOx})}{\partial X}$	$(P^{B_{CO_2}}, P^{B_{SO_2}}, P^{B_{NOx}})$
(1, 0, 0)	(2.0273, 0, 0)	(61.41, N/A, N/A)
(0.9, 0.1, 0)	(2.0060, 0.0008, 0)	(61.47, 146485, N/A)
(0.8, 0.2, 0)	(1.9755, 0.0019, 0)	(61.67, 65317, N/A)
(0.7, 0.3, 0)	(1.9615, 0.0032, 0)	(61.97, 38288, N/A)
(0.6, 0.4, 0)	(1.9545, 0.0049, 0)	(62.37, 24774, N/A)
(0.5, 0.5, 0)	(1.9502, 0.0074, 0)	(62.94, 16666, N/A)
(0.4, 0.6, 0)	(1.9472, 0.0110, 0)	(63.79, 11261, N/A)
(0.3, 0.7, 0)	(1.9450, 0.0171, 0)	(65.21, 7400, N/A)
(0.2, 0.8, 0)	(1.6151, 0.0244, 0)	(68.04, 4504, N/A)
(0.1, 0.9, 0)	(1.0208, 0.0294, 0)	(78.94, 2322, N/A)
(0, 1, 0)	(0, 0.0718, 0)	(N/A, 1167, N/A)

Individual Shadow Price (ISP)



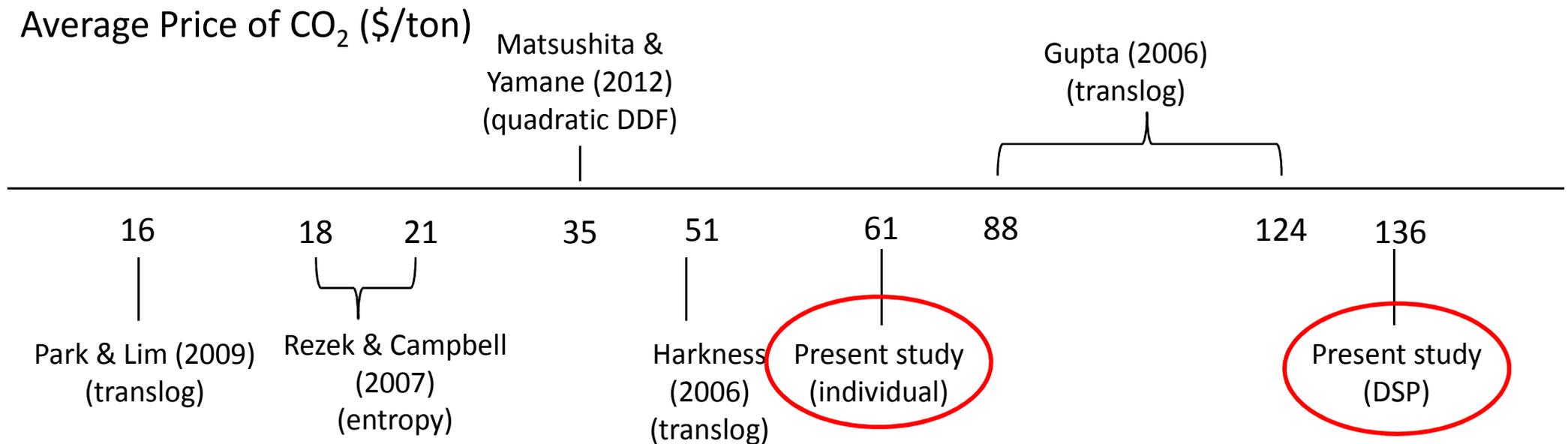
# Empirical Study

- Direction Generation ( $g^{B_{CO_2}}, g^{B_{SO_2}}, g^{B_{NOx}}$ )
  - Through a literature-based method, we found that the direction  $(g^{B_{CO_2}}, g^{B_{SO_2}}, g^{B_{NOx}}) = (0.048, 0.508, 0.444)$ .

Method		Literature-based			Method		Individual-SP		
Direction ( $g^{B_{CO_2}}, g^{B_{SO_2}}, g^{B_{NOx}}$ )		0.048	0.508	0.444	Direction ( $g^{B_{CO_2}}, g^{B_{SO_2}}, g^{B_{NOx}}$ )		0.045	0.631	0.324
Boyd	DSP	128	3212	17983	DSP	131	2478	23615	
et al. (1996)	Benchmarking Ratio (DSP/MSPL)	1.03	1.03	1.03	Benchmarking Ratio (DSP/ISP)	1.85	1.838	2.16	
Turner	DSP	125	3121	17470	DSP	128	2410	22965	
(1995)	Benchmarking Ratio (DSP/MSPL)	1.01	1.01	1.01	Benchmarking Ratio (DSP/ISP)	2.06	1.82	3.04	

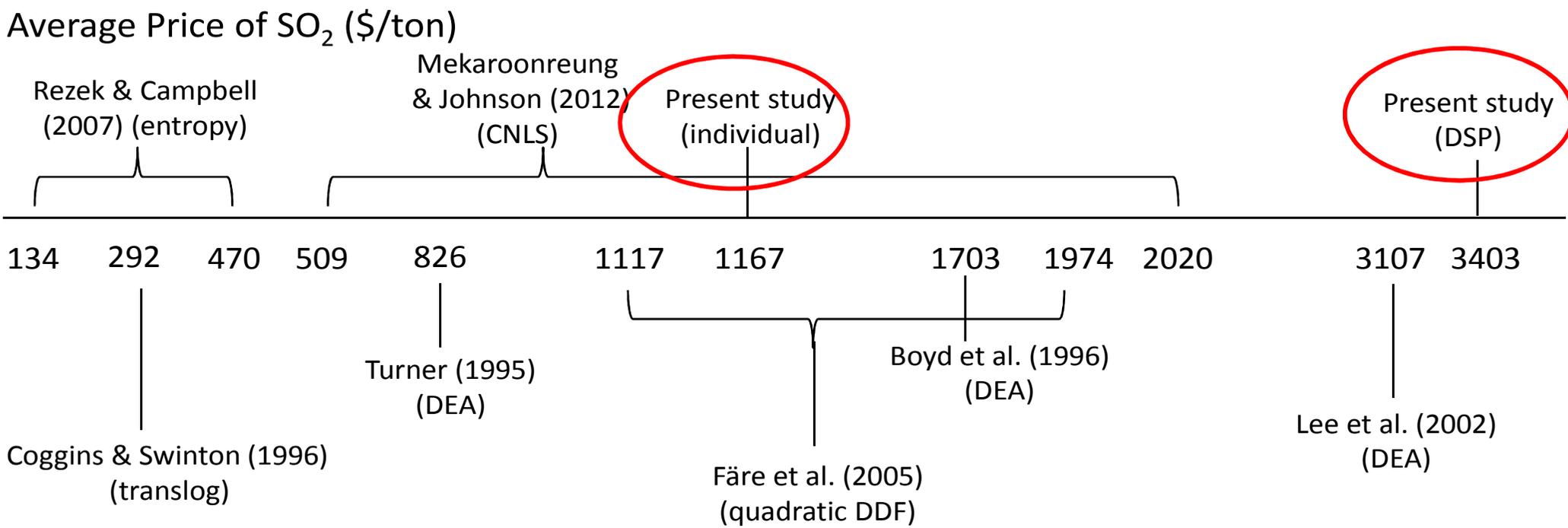
# Empirical Study

- Comparison of studies for shadow price estimations in electric power sectors



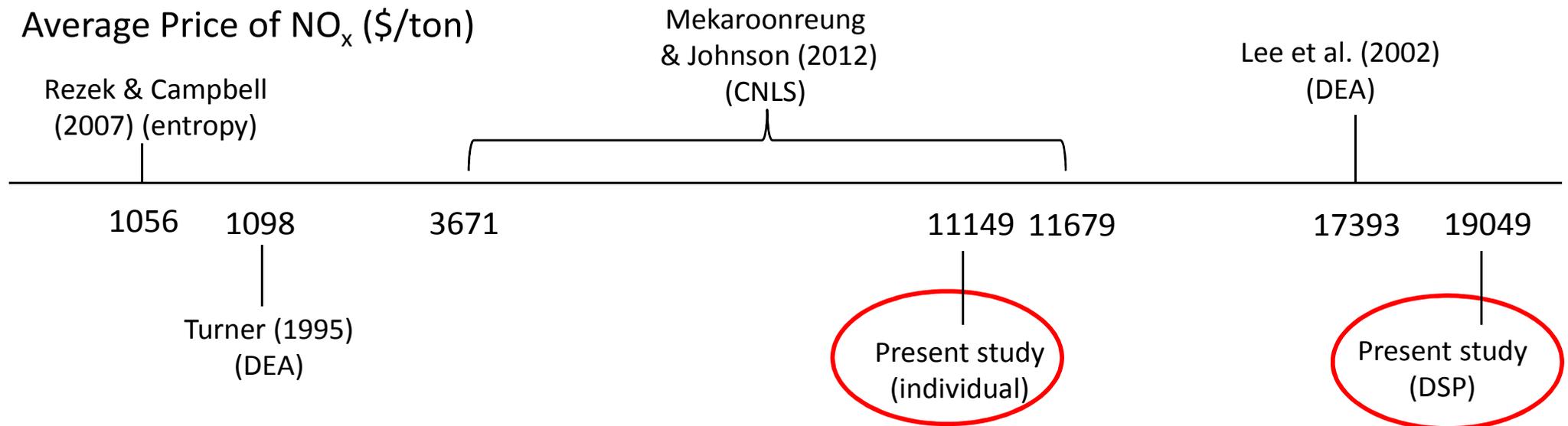
# Empirical Study

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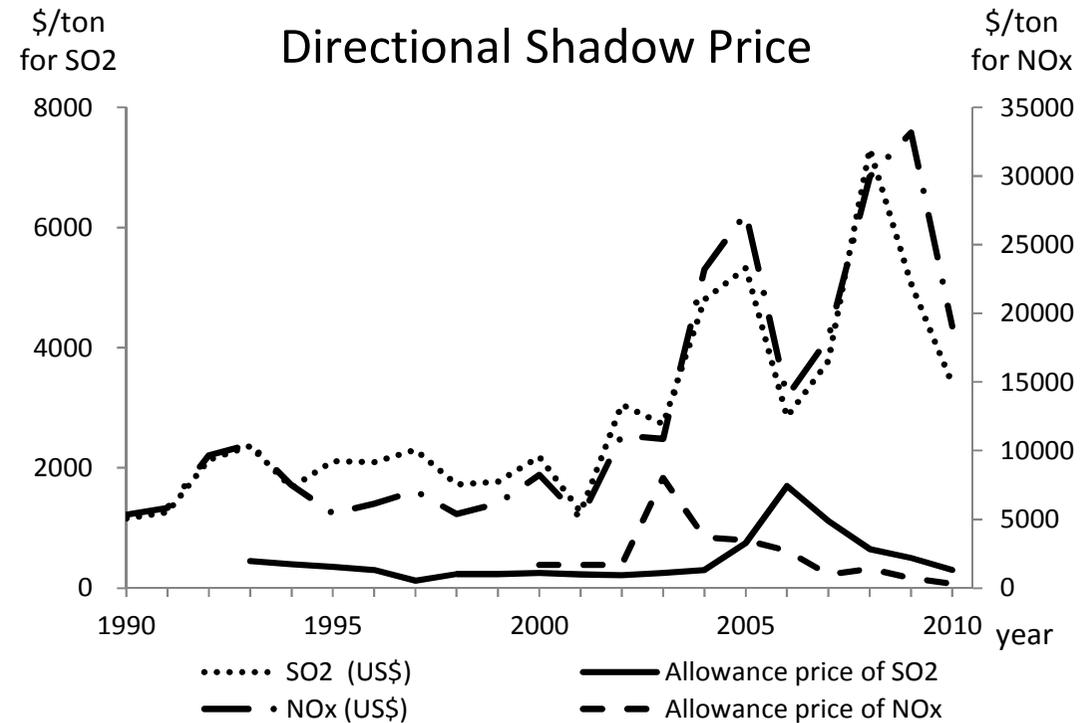
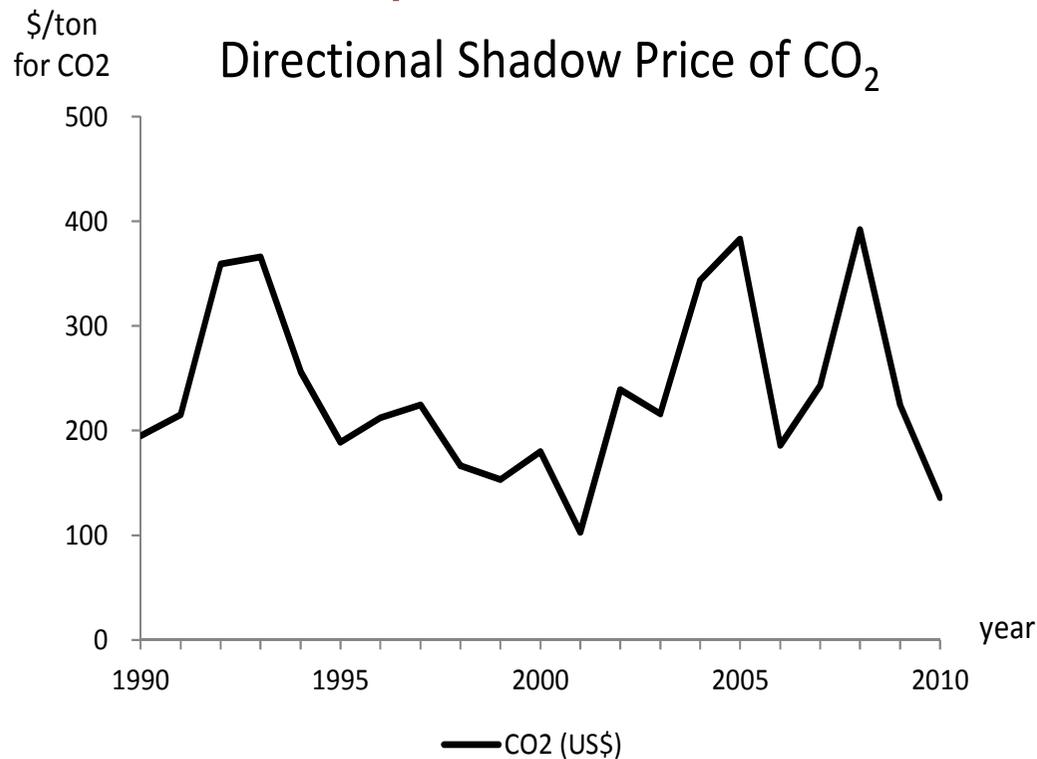
# Empirical Study

- Comparison of studies for shadow price estimations in electric power sectors



# Empirical Study

## • DSP of pollutants in 1990-2010



- The DSPs of SO<sub>2</sub> and NO<sub>x</sub> rise gradually due to a successful emission reduction and [Clean Air Interstate Rule \(CAIR\)](#) beginning in 2010.
- Allowance prices rose in 2003 & 2006 since CAIR provided incentives for utilities to purchase allowances and bank them [for future use](#).
- After 2005, emission levels fell because of the increased use of [gas-fired boilers](#) and pollution control equipment. Thus, an [excess supply](#) of allowances in the market caused allowance market prices to fall.

# Conclusions

- **Theoretical Benefits**

- Directional marginal product
- Directional shadow price estimation of bad outputs
- Shadow price substitution of bad outputs
- Comparison of previous studies: addressing the **issue**
  - estimating shadow prices separately may lead to an **overestimation** of marginal productivity and an **underestimation** of shadow price

- **Practical Benefits**

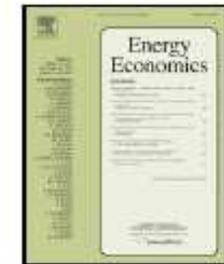
- Provide environmental policy guidelines
  - the allowance price in emission trading markets
- Bargaining → **Reasonable** marginal abatement cost (MAC)
- Support Cap-and-Trade regulation



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## Energy Economics

journal homepage: [www.elsevier.com/locate/eneco](http://www.elsevier.com/locate/eneco)



### Directional shadow price estimation of CO<sub>2</sub>, SO<sub>2</sub> and NO<sub>x</sub> in the United States coal power industry 1990–2010



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#### ABSTRACT

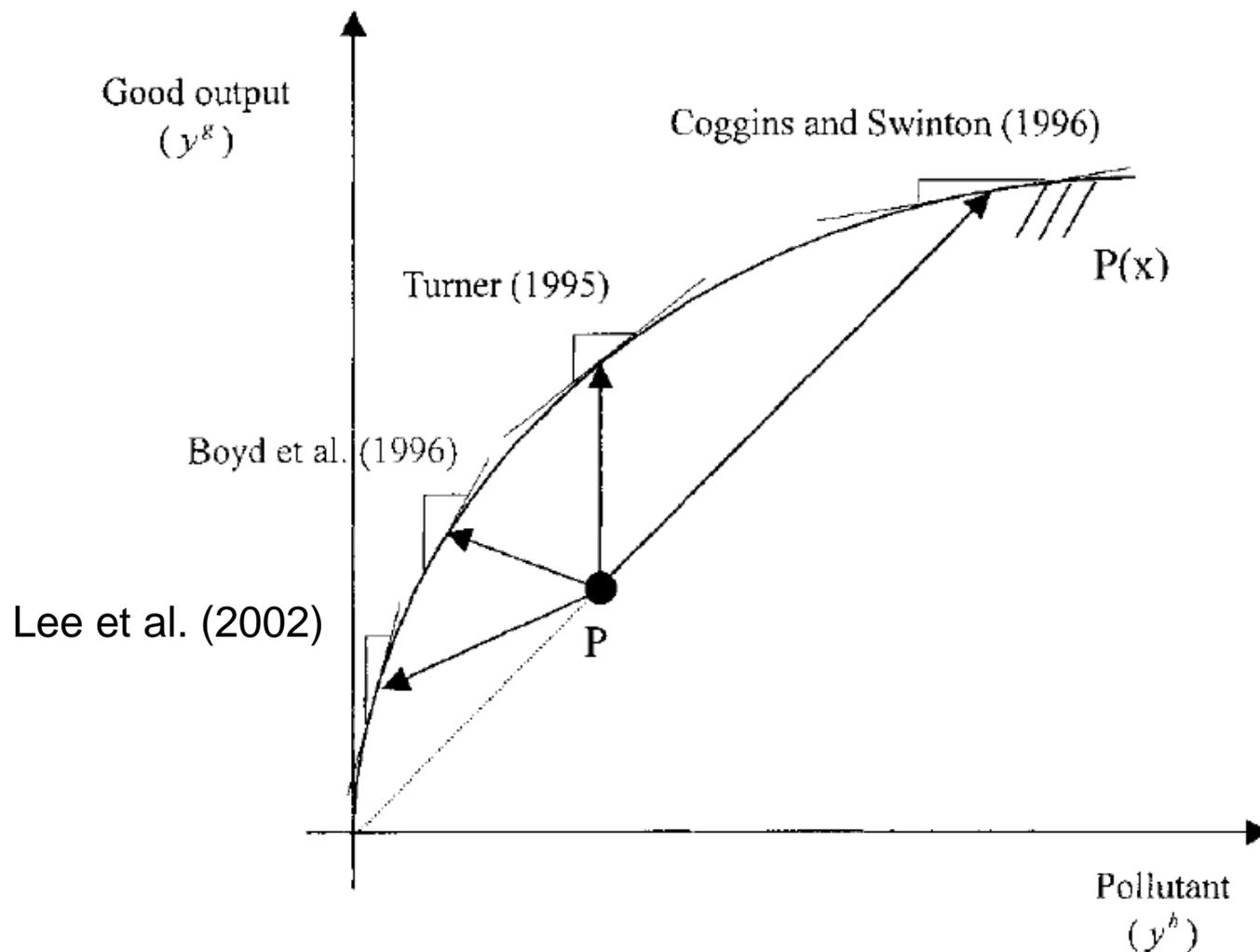
Shadow prices, also termed marginal abatement costs, provide valuable guidelines to support environmental regulatory policies for CO<sub>2</sub>, SO<sub>2</sub> and NO<sub>x</sub>, the key contributors to climate change. This paper complements the existing models and describes a directional marginal productivity (DMP) approach to estimate directional shadow prices (DSPs) which present substitutability among three emissions and are jointly estimated. We apply the method to a case study of CO<sub>2</sub>, SO<sub>2</sub> and NO<sub>x</sub> produced by coal power plants operating between 1990 and 2010 in the United States. We find that DSP shows 1.1 times the maximal shadow prices estimated in the current literature. We conclude that estimating the shadow prices of each by-product separately may lead to an overestimation of the marginal productivity and an underestimation of the shadow prices.

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Any other issues in MAC  
estimation?

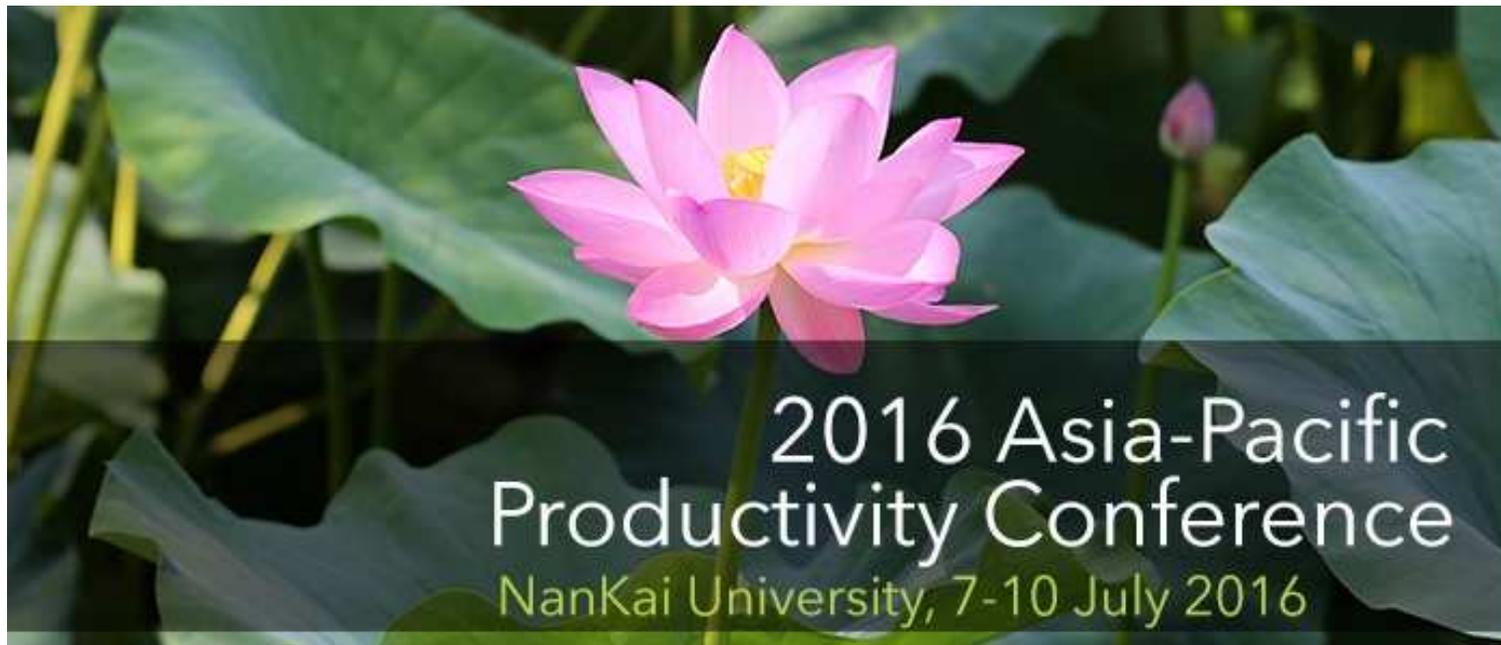
# Remaining Issues...

- Issue1: **direction** used for projecting inefficient firms to frontier will **significantly** affect the MAC estimation (Lee et al. 2002; Zhou et al. 2014)
- Issue2: **average** or weighted average of multiple firm-specific MACs



# “Nash Marginal Abatement Costs!”

see you in APPC2016...



Thanks for your attention!

Q&A

