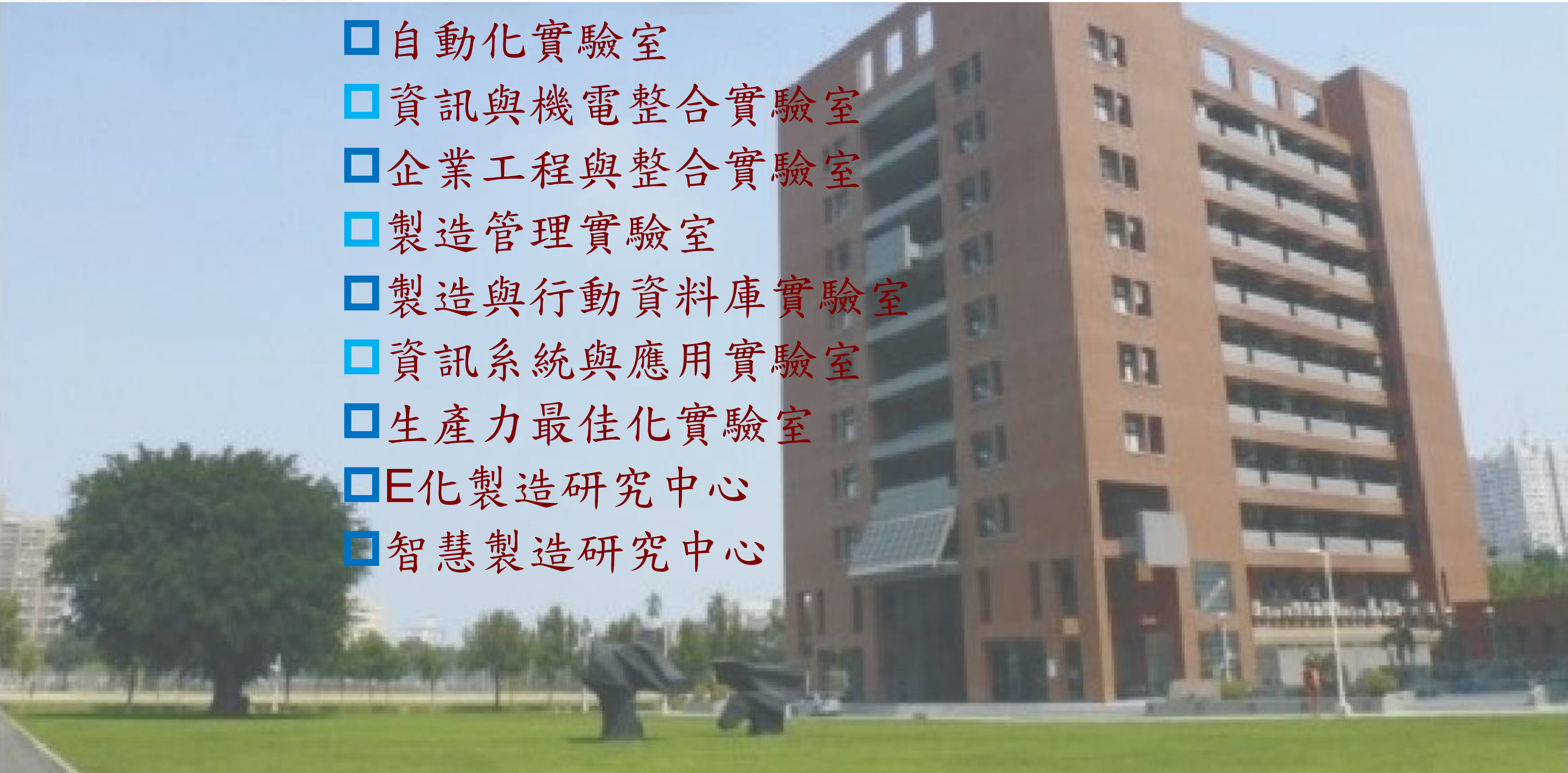
# 混合互補問題與納許均衡
# (Mixed Complementarity Problem and Nash Equilibrium)

Dr. Chia-Yen Lee (李家岩 博士)

2019/08/16

Institute of Manufacturing Information and Systems (製造資訊與系統研究所)
Dept. of Computer Science and Information Engineering (資訊工程系)
Engineering Management Graduate Program (工程管理碩士在職專班)
National Cheng Kung University (國立成功大學)

成功大學製造資訊與系統研究所(前稱製造工程研究所)，成立於民國八十三年八月，任務在於配合我國高科技產業發展及傳統工業升級之需要，培養具E化製造、製造技術、製造系統、製造知識管理與應用及製造管理之製造資訊與系統整合人才，並以執行產學合作計畫之方式，提昇產業競爭力之製造資訊系統研究。



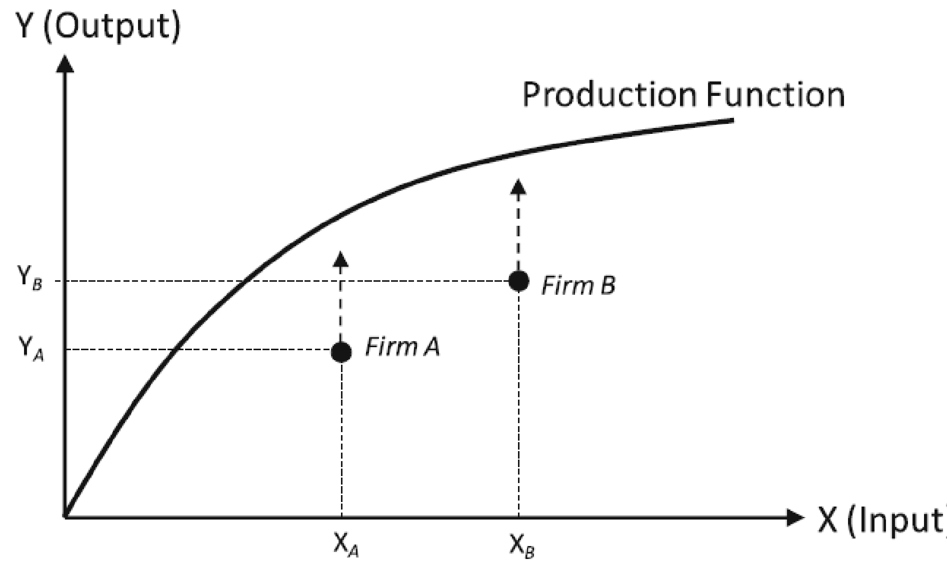
- 自動化實驗室
- 資訊與機電整合實驗室
- 企業工程與整合實驗室
- 製造管理實驗室
- 製造與行動資料庫實驗室
- 資訊系統與應用實驗室
- 生產力最佳化實驗室
- E化製造研究中心
- 智慧製造研究中心

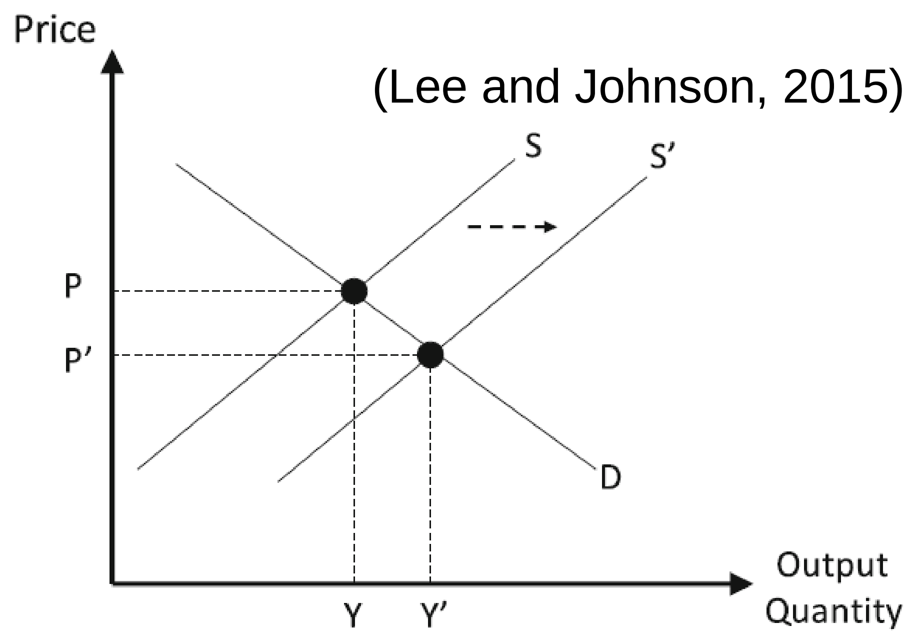
# Outline

- Introduction
- Nash Equilibrium Identified in DEA
- Mixed Complementarity Problem (MiCP)
- Proposed Models
  - Case1: Nash-Profit Efficiency Measuring Market Structures
  - Case2: Mixed-Strategy Nash Equilibrium
  - Case3: Nash Shadow Price Estimation
  - Case4: Allocation of Emission Permits
- Conclusions

# Introduction

- Typical Assumption in DEA
  - All firms want to be as productive as possible
  - An exogenous price in a perfectly competitive market (Cherchye, Kuosmanen, & Post, 2002)
  - DMU are independent with each other




(Lee and Johnson, 2015)

# Introduction

## Imperfectly Competitive Market

- An inefficient firm that increases output may reduce overall profits by increasing the market quantity and causing the market price to fall (Johnson and Ruggiero, 2011).

- Rational Inefficiency: A firm is maximizing its profit by intentionally operating at lower productivity levels (Lee and Johnson, 2015)
  - non-cooperative game (Nash, 1951) (Lee & Johnson, 2015)
  - a firm may overestimate the revenue when expanding output by assuming exogenous price (Lee, 2016).

- Energy market typically is imperfectly competitive
  - market price can be affected by the total supply which is generated by all firms in the market. That is, the market price is endogenous (Hobbs and Pang, 2007; Gabriel, et al., 2013; Lee and Johnson, 2015; Lee, 2016).

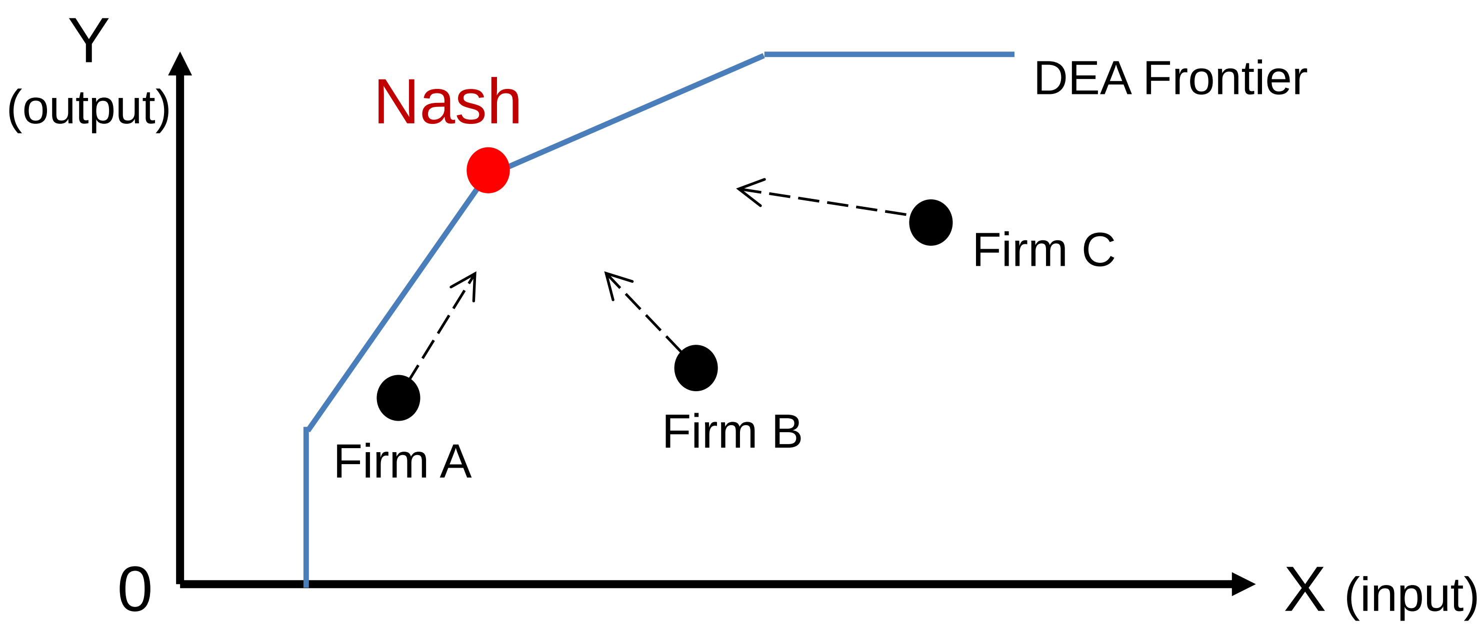
# Introduction

## DEA Games

- The seminal works of the unconstrained and constrained two-person zero-sum games were developed by Banker (Banker, 1980)
- Aparico et al. (J. Aparicio, Landete, Monge, & Sirvent, 2008) proposed the iterative multi-unit combinatorial auctions based on a linear anonymous pricing scheme. They emphasized on determining the price of any bundle of items through a computational experiment.
- Wu et al. (Wu, Liang, Yang, & Yan, 2009) proposed the Nash bargaining game to improve the non-uniqueness and average properties of the cross-efficiency measure.
- Lozano (Lozano, 2013) extended the linear production game to a general production function and formulated a DEA production collaborative game. The full-cooperation scenario and the partial-cooperation scenario are investigated
- However, some unsolved issues remain in the literature.
  - Endogenous price in imperfectly competitive market
  - Solve several firm-specific profit maximization models in one shot

## Nash Equilibrium

- Nash equilibrium is a solution of a **non-cooperative** game involving two or more players, and **no player has incentive** to change the strategy due to a reduction in the immediate payoff.
- In the imperfectly competitive markets, the inverse demand function of desirable output $P_j^Y(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}) = P_j^{Y_0} - \alpha_{jj}\bar{Y}_j - \sum_{h \neq j} \alpha_{jh}\bar{Y}_h$ , where $\bar{Y}_j = \sum_{k \neq r} y_{kj} + y_{rj}$, $\bar{\boldsymbol{Y}}_{(-j)} = \{\bar{Y}_1, \dots, \bar{Y}_{j-1}, \bar{Y}_{j+1}, \dots, \bar{Y}_{|J|}\}$. (Nash-Cournot Game)



# Nash Equilibrium Identified on DEA Frontier

POLab

## Nash Profit Function (for each firm $r$)

- $$NPF_r^* = \max_{y_{rj},x_{ri}} \left\{ \sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ri} \,\middle|\, \begin{array}{c} \sum_k \lambda_{rk} y_{kj} \geq y_{rj}, \forall j; \\ \sum_k \lambda_{rk} x_{ki} \leq x_{ri}, \forall i; \\ \sum_k \lambda_{rk} = 1; \\ \lambda_{rk} \geq 0, \forall k; \end{array} \right\}$$
- where $\bar{Y}_j = \sum_{k \neq r} y_{kj} + y_{rj}$, $\bar{\boldsymbol{Y}}_{(-j)} = \{\bar{Y}_1, \ldots, \bar{Y}_{j-1}, \bar{Y}_{j+1}, \ldots, \bar{Y}_{|J|}\}$ and output price function $P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) = P_j^{Y_0} - \alpha_{jj}\bar{Y}_j - \sum_{h \neq j} \alpha_{jh}\bar{Y}_h$.

## Nash Equilibrium

**Definition**: Let $K$ be a finite number of players, $\theta$ a utility (or profit) function, $T_k$ a strategy set (production possibility set) for player $k = 1, \ldots, |K|$, and $(\boldsymbol{x}_k, \boldsymbol{y}_k) = \left(x_{k1}, \ldots, x_{k|I|}, y_{k1}, \ldots, y_{k|J|}\right) \in T_k$ an observed production vector. A vector $(\boldsymbol{x}^*, \boldsymbol{y}^*) = \left((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \ldots, (\boldsymbol{x}_{|K|}^*, \boldsymbol{y}_{|K|}^*)\right) \in T_1 \times T_2 \times \cdots \times T_{|K|}$ is called a Nash equilibrium if and only if

$$\theta(\boldsymbol{x}^*, \boldsymbol{y}^*) \geq \theta\left(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_{(-k)}^*, \boldsymbol{y}_k, \hat{\boldsymbol{y}}_{(-k)}^*\right), \forall (\boldsymbol{x}_k, \boldsymbol{y}_k) \in T_k,$$

where $\hat{\boldsymbol{x}}_{(-k)}^* = (\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_{k-1}^*, \boldsymbol{x}_{k+1}^*, \ldots, \boldsymbol{x}_{|K|}^*)$ and $\hat{\boldsymbol{y}}_{(-k)}^* = (\boldsymbol{y}_1^*, \ldots, \boldsymbol{y}_{k-1}^*, \boldsymbol{y}_{k+1}^*, \ldots, \boldsymbol{y}_{|K|}^*)$ holds for all $k = 1, \ldots, |K|$.

## NPF is a concave function? (Lee and Johnson, 2015)

- strictly concave to verify the existence and uniqueness
- $NPF_r = \sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right)y_{rq} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right)x_{ri}$ is continuous and differentiable almost everywhere on $(\boldsymbol{x}, \boldsymbol{y}) \in \tilde{T}$.
- That is, the revenue function $\sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right)y_{rj}$ should be strictly concave and the cost function $\sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right)x_{ri}$ strictly convex.
- We have $\frac{\partial NPF_r}{\partial y_{rj}} = P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) - \alpha_{jj}y_{rj} - \sum_{h\neq j}\alpha_{hj}y_{rh}$, and $\frac{\partial^2 NPF_r}{\partial y_{rj}\partial y_{rh}} = -\alpha_{jh} - \alpha_{hj}$. A negative definite Hessian matrix will imply a strictly concave revenue function.

## Diagonal Dominance Property

- The necessary and sufficient conditions are $\alpha_{jh} > 0$ and the price sensitivity matrix $\boldsymbol{\alpha}$ satisfies the diagonal dominance property, namely, $\alpha_{jj} > \sum_{h\neq j}\alpha_{jh}$ for all output $j$.

# Nash Equilibrium Identified on DEA Frontier

- Each firm maximizes its Nash profit function
  - How to solve N optimization models in one shot?

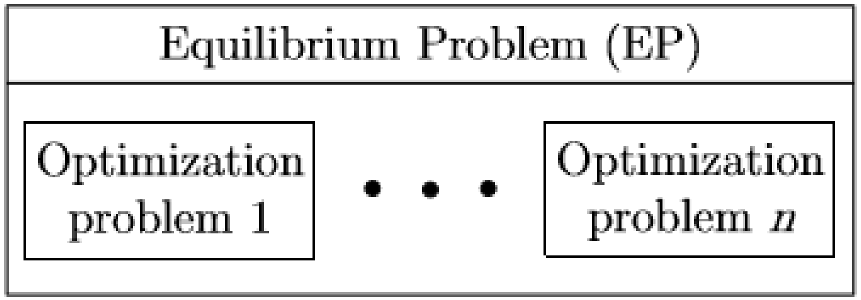
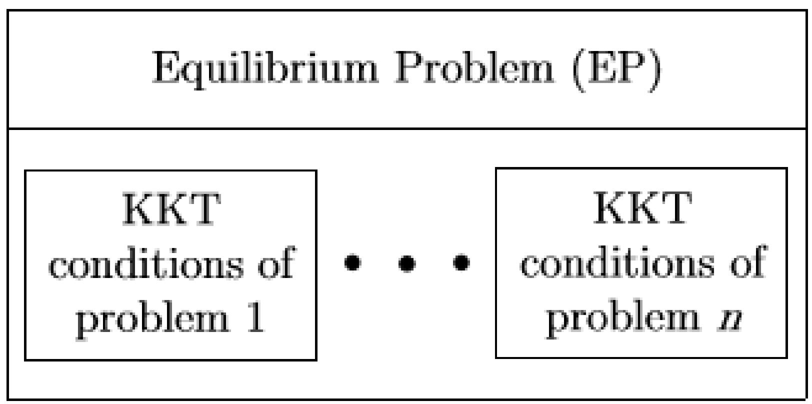



**Fig. 2.3** Equilibrium problem: Joint solution of a number of interrelated optimization problems




Gabriel, et al. (2013)

**Fig. 2.4** Equilibrium problem (KKT): Joint solution of several systems of KKT conditions

- Karamardian (1971) proves that each generalized complementarity problem, i.e., KKT condition, corresponds to a set of variational inequality.

**Theorem**: Consider an imperfectly competitive market with $K$ firms, an inverse demand function $P^Y(\cdot)$ that is strictly decreasing and continuously differentiable in $y$, and an inverse supply function $P^X(\cdot)$ that is strictly increasing and continuously differentiable in $x$. Since Lemma 2.1 shows that the profit function $\theta_k(x_k, y_k)$ is concave and the variables $x_k, y_k \geq 0$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*) = ((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \dots, (\boldsymbol{x}_K^*, \boldsymbol{y}_K^*))$ is a Nash equilibrium solution if and only if

$\nabla_{\boldsymbol{x}_k} \theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0$ and $\nabla_{\boldsymbol{y}_k} \theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq 0, \forall k$;

$\boldsymbol{x}_k^* [\nabla_{\boldsymbol{x}_k} \theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)] = 0$ and $\boldsymbol{y}_k^* [\nabla_{\boldsymbol{y}_k} \theta_k(\boldsymbol{x}^*, \boldsymbol{y}^*)] = 0, \forall k$,

where $(\boldsymbol{x}_k^*, \boldsymbol{y}_k^*) \in \tilde{T}$.

POLab

## Lagrangian function

- $L_r(y_{rj}, x_{ri}, \lambda_{rk}, \mu_{rj}, \nu_{ri}, \xi_r) = \sum_j P_j^Y(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}) y_{rj} - \sum_i P_i^X(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}) x_{ri} - \sum_j \mu_{rj}\left(y_{rj} - \sum_k \lambda_{rk}\, y_{kj}\right) - \sum_i \nu_{ri}\left(\sum_k \lambda_{rk}\, x_{ki} - x_{ri}\right) - \xi_r\left(\sum_k \lambda_{rk} - 1\right)$

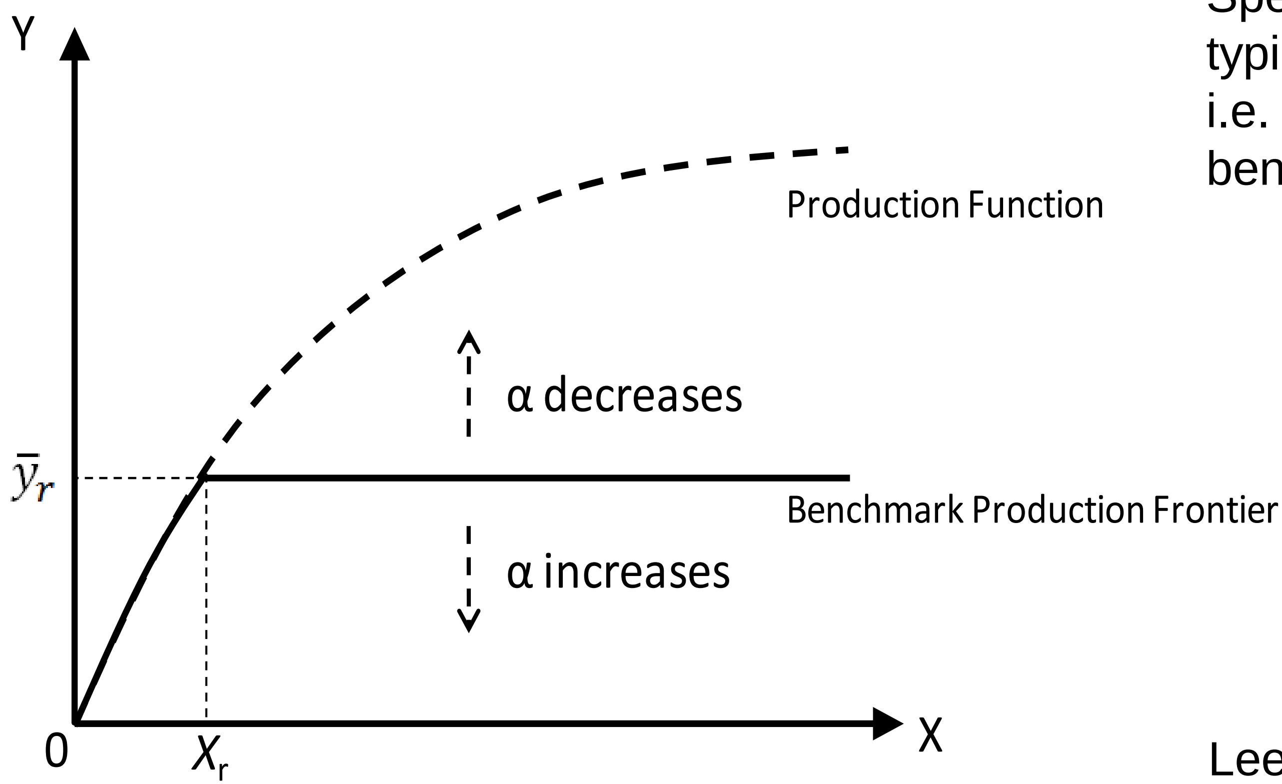
## Mixed Complementarity Problem (MiCP)

orthogonal

- $0 \geq \frac{\partial L_r}{\partial y_{rj}} = \left(P_j^Y(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}) - \alpha_{jj} y_{rj} - \sum_{h \neq j} \alpha_{hj} y_{rh} - \mu_{rj}\right) \perp y_{rj} \geq 0, \quad \forall r, j$
- $0 \geq \frac{\partial L_r}{\partial x_{ri}} = \left(-P_i^X(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}) - \beta_{ii} x_{ri} - \sum_{l \neq i} \beta_{li} x_{rl} + \nu_{ri}\right) \perp x_{ri} \geq 0, \quad \forall r, i$
- $0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_j \mu_{rj} y_{kj} - \sum_i \nu_{ri} x_{ki} - \xi_r\right) \perp \lambda_{rk} \geq 0, \quad \forall r, k$
- $0 \geq \left(y_{rj} - \sum_k \lambda_{rk}\, y_{kj}\right) \perp \mu_{rj} \geq 0, \quad \forall r, j$
- $0 \geq \left(\sum_k \lambda_{rk}\, x_{ki} - x_{ri}\right) \perp \nu_{ri} \geq 0, \quad \forall r, i$
- $0 = \left(\sum_k \lambda_{rk} - 1\right), \quad \forall r$

# Nash Equilibrium Identified on DEA Frontier

**Corollary**: For single output case, If $P(\bar{Y}) = P^0 - \alpha\bar{Y} \geq 0$ and α is a small enough positive parameter, then the Nash equilibrium solution is for all firms to produce on the production frontier. Otherwise, if α is a large enough positive parameter, then the MiCP will lead to a benchmark output level with $y_r = \bar{y_r}$ close to zero, where $\bar{y_r}$ defines a truncated output level.




Special case: if $\alpha = 0$, then typical profit maximization. i.e. allocative efficient benchmark

Lee and Johnson (2015)

## Market Structure (Lee, 2016, EJOR)

- Nash profit efficiency
- New York State's gas and oil industry

## Mixed-Strategy Equilibrium (Lee, 2018, EJOR)

- Multi-oriented efficiency measure
- China's regional electric power industry sectors in 2010

## Shadow Price of Pollution (Lee & Wang, 2019, EJOR)

- Directional shadow price (DSP) estimation on StoNED frontier
- 2013 China Coal Power Plants in North and Northeast regions

## Allocation of Emission Permits (Lee, 2019, JEM)

- Efficiency estimation before and after AEP
- Coal-fired power plants operating in China in 2013

# Introduction

- Energy Market with Undesirable Output (Dakpo et al., 2016)
  - (1) treating the pollution as a free disposable input (Atakelty Hailu & Veeman, 2001), but challenged as it violates the physical laws (Färe & Grosskopf, 2003)
  - (2) data transformation applied to treat the bad outputs as good outputs equivalently (Seiford & Zhu, 2002), but challenged due to undesirable output reduction without any cost (Färe & Grosskopf, 2004)
  - (3) assuming the weak disposability and nulljointness of good outputs and bad outputs (Färe, Grosskopf, Lovell, & Pasurka, 1989) (Färe & Grosskopf, 2009), but violating the law of thermodynamics (Coelli, Lauwers, & Van Huylenbroeck, 2007)
  - (4) the material balance principles requiring knowledge of the technical coefficients between desirable outputs, undesirable outputs and inputs (Hampf & Rødseth, 2014)
  - (5) the use of two sub-technologies (i.e., by-production) (Murty, Robert Russell, & Levkoff, 2012).

# Nash-Profit Efficiency for Market Structure

Lee, Chia-Yen, 2016. Nash-Profit Efficiency: A Measure of Changes in Market Structures. European Journal of Operational Research, 255 (2), 659-663.

# Nash-Profit Efficiency for Market Structure

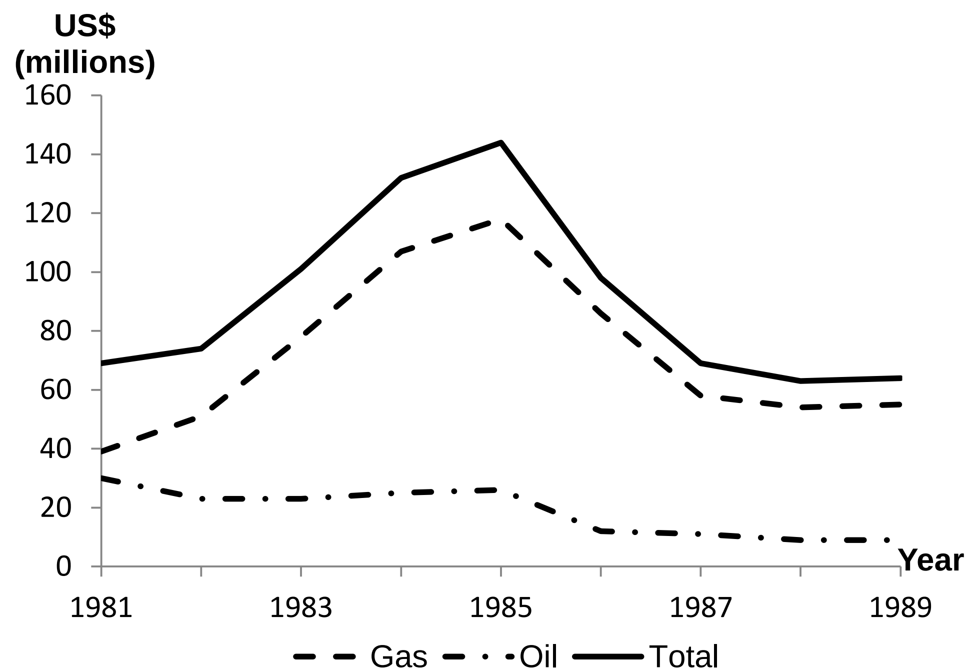
- Background and Motivation
- Nash Equilibrium in Production Possibility Set
- Directional Nash Technical Efficiency Estimator
- Nash Profit Efficiency and its Decomposition
- Empirical Study- Natural Gas and Oil Market of New York State in 1980s
- Conclusions

#

# Nash-Profit Efficiency for Market Structure

## Background and Motivation

- New York State's steady increase in annual production since the mid-1970's is attributed to OPEC's oil embargo in 1973 and the ensuing global price shocks.
- Passage of the Federal Natural Gas Policy Act of 1978 accelerated natural gas production nationally, in 1986 an oil glut caused by oil price deregulation in 1981 was similar to the effect of natural gas deregulation in 1986.
- A slowdown in drilling activity in New York State occurred between 1985 and 1988, mostly as a result of the falling prices for oil and natural gas.
  - Gas: Average wellhead prices $3.37 in 1985 to $2.3 in 1988 per thousand cubic feet
  - Oil: $25.19 to $14.97 per barrel




Market value of New York State's oil and gas production, 1981–1989

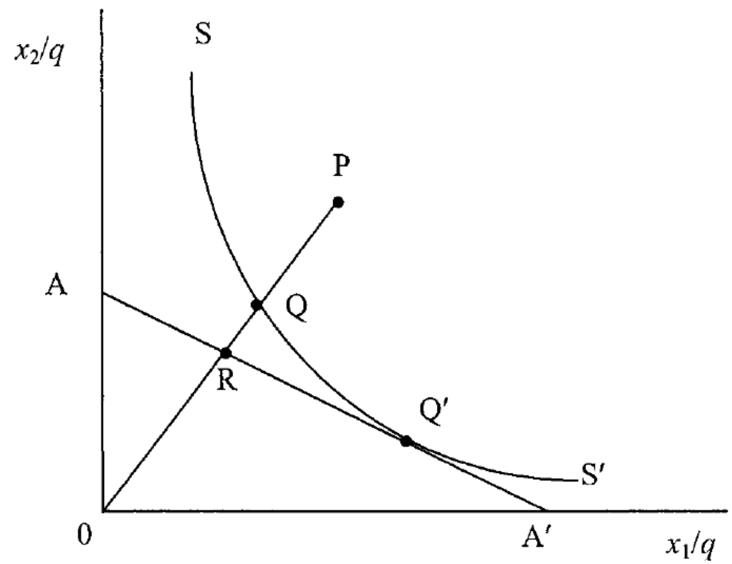
# Nash-Profit Efficiency for Market Structure

- Literature Review
- Benefits of understanding the market structure (Nickell, 1996).
  - a firm wants to know how its competition affects efficiency and productivity growth in order to devise survival strategies in different market structures
  - since the intensity of competition is not independent of firm behavior, a high-performing firm may gain market power in the long run
- Price cost margin (PCM), eg. Lerner Index
  - however, more intense competition leads to higher PCM instead of lower margins (Boone, 2008)
- Concentration ratio (CR), eg. $CR_4$ (total market share of the 4 largest firms)
  - a ratio of less than 40% is considered competitive, and a ratio of more than 40% is considered an oligopoly (Bain, 1951)
  - however, only represents the "relative scale size" of a firm without a price premium
  - an industry having potentially thousands of firms and does not reveal the different distributions in market share among the top 4 companies
  - competition measured by increased numbers of competitors is associated with a significantly higher rate of productivity growth (Nickell, 1996). In other words, both productivity and market share drive firm-specific market power

# Nash-Profit Efficiency for Market Structure

- Productivity analysis attributes the entire difference between the production frontier and the observation to operations in two dimensions–technical efficiency and allocative efficiency (Nerlove, 1965).

- Technical inefficiency is attributed to poor engineering practice and allocative inefficiency to economic mismanagement (Zofio et al., 2013).




Coelli et al. (2005)

# Nash-Profit Efficiency for Market Structure

- However, three issues need to be addressed.
  - How to determine the "orientation" for efficiency estimation (Färe et al., 2013)? → Nash equilibrium direction in imperfectly competitive market
  - In imperfectly competitive market, using exogenous price to calculate the efficiency is a bias since each firm can control the output level affecting the market price (Lee and Johnson, 2015)? → endogenous price
  - Typical CR measuring market structure shows some flaws and does not consider the productivity perspective (since both productivity and market share drive firm-specific market power). CR cannot provide the suggestion to drive productivity. → Nash profit efficiency

# Nash-Profit Efficiency for Market Structure

## Research Objective

- Present an alternative direction (i.e., orientation) towards the Nash equilibrium used in DDF (Chambers et al., 1996) for efficiency estimation.

- In this case, the Nash equilibrium provides a profit-maximizing allocative efficient benchmark in an imperfectly competitive market, even though a set of firms will choose not to produce on the production frontier.

- Propose a new index to measure the change of market structure in Natural Gas and Oil Market of New York State in 1980s

# Nash-Profit Efficiency for Market Structure

## Nash Equilibrium in Production Possibility Set

$$\tilde{T} = \left\{ (\boldsymbol{x}, \boldsymbol{y}) \middle| \begin{array}{c} \sum_k \lambda_k y_{kj} \geq y_j, \forall j; \\ \sum_k \lambda_k x_{ki} \leq x_i, \forall i; \\ \sum_k \lambda_k = 1; \\ \lambda_k \geq 0, \forall k; \end{array} \right\}$$

## Nash Equilibrium

**Definition 1**: Let $K$ be a finite number of players, $\theta$ a utility (or profit) function, $T_k$ a strategy set (production possibility set) for player $k = 1, \dots, |K|$ , and $(\boldsymbol{x}_k, \boldsymbol{y}_k) = (x_{k1}, \dots, x_{k|I|}, y_{k1}, \dots, y_{k|J|}) \in T_k$ an observed production vector. A vector $(\boldsymbol{x}^*, \boldsymbol{y}^*) = \left((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*), \dots, (\boldsymbol{x}_{|K|}^*, \boldsymbol{y}_{|K|}^*)\right) \in T_1 \times T_2 \times \cdots \times T_{|K|}$ is called a Nash equilibrium if and only if

$$\theta(\boldsymbol{x}^*, \boldsymbol{y}^*) \geq \theta\left(\boldsymbol{x}_k, \widehat{\boldsymbol{x}}_{(-k)}^*, \boldsymbol{y}_k, \widehat{\boldsymbol{y}}_{(-k)}^*\right), \forall (\boldsymbol{x}_k, \boldsymbol{y}_k) \in T_k,$$

where $\widehat{\boldsymbol{x}}_{(-k)}^* = (\boldsymbol{x}_1^*, \dots, \boldsymbol{x}_{k-1}^*, \boldsymbol{x}_{k+1}^*, \dots, \boldsymbol{x}_{|K|}^*)$ and $\widehat{\boldsymbol{y}}_{(-k)}^* = (\boldsymbol{y}_1^*, \dots, \boldsymbol{y}_{k-1}^*, \boldsymbol{y}_{k+1}^*, \dots, \boldsymbol{y}_{|K|}^*)$ holds for all $k = 1, \dots, |K|$.

POLab

## Profit Maximization Function

$$NPF_r^* = \max_{y_{rj},x_{ri}} \left\{ \sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ri} \;\middle|\; \begin{array}{c} \sum_k \lambda_{rk}\, y_{kj} \geq y_{rj}, \forall j; \\ \sum_k \lambda_{rk}\, x_{ki} \leq x_{ri}, \forall i; \\ \sum_k \lambda_{rk} = 1; \\ \lambda_{rk} \geq 0, \forall k; \end{array} \right\}$$

- where $\bar{Y}_j = \sum_{k\neq r} y_{kj} + y_{rj}$, $\bar{\boldsymbol{Y}}_{(-j)} = \{\bar{Y}_1, \ldots, \bar{Y}_{j-1}, \bar{Y}_{j+1}, \ldots, \bar{Y}_{|J|}\}$ and output price function $P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) = P_j^{Y_0} - \alpha_{jj}\bar{Y}_j - \sum_{h\neq j} \alpha_{jh}\bar{Y}_h$.

**Lemma 1**: Given the price function of input and output defined above, if matrix $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ satisfy "*diagonal dominance*", then the profit function $\sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ri}$ is strictly concave.

## Mixed Complementarity Problem (MiCP)

$$0 \geq \frac{\partial L_r}{\partial y_{rj}} = \left(P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) - \alpha_{jj} y_{rj} - \sum_{h \neq j} \alpha_{hj} y_{rh} - \mu_{rj}\right) \perp y_{rj} \geq 0, \quad \forall r, j$$

$$0 \geq \frac{\partial L_r}{\partial x_{ri}} = \left(-P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) - \beta_{ii} x_{ri} - \sum_{l \neq i} \beta_{li} x_{rl} + \nu_{ri}\right) \perp x_{ri} \geq 0, \quad \forall r, i$$

$$0 \geq \frac{\partial L_r}{\partial \lambda_{rk}} = \left(\sum_j \mu_{rj} y_{kj} - \sum_i \nu_{ri} x_{ki} - \xi_r\right) \perp \lambda_{rk} \geq 0, \quad \forall r, k$$

$$0 \geq \left(y_{rj} - \sum_k \lambda_{rk} y_{kj}\right) \perp \mu_{rj} \geq 0, \quad \forall r, j$$

$$0 \geq \left(\sum_k \lambda_{rk} x_{ki} - x_{ri}\right) \perp \nu_{ri} \geq 0, \quad \forall r, i$$

$$0 = \left(\sum_k \lambda_{rk} - 1\right), \quad \forall r$$

**Theorem 1**: If the profit function is strictly concave, then the proposed MiCP generates an efficient Nash equilibrium $(x_{ri}, y_{rj}) \in \tilde{T}$ and is unique for the profit maximization problem.

# Nash-Profit Efficiency for Market Structure

POLab

- Directional Nash Technical Efficiency Estimator
- Directional Distance Function

$$\vec{D}_{\tilde{T}}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g_x}, \boldsymbol{g_y}) = \max\{\gamma | (\boldsymbol{x} - \gamma \boldsymbol{g_x}, \boldsymbol{y} + \gamma \boldsymbol{g_y}) \in \tilde{T}\}$$

- Direction Vector

| Study | Name | Direction Vector $(\boldsymbol{g_x}, \boldsymbol{g_y})$ | Economic Implications | Price Info. |
|---|---|---|---|---|
| **Färe et al. (2006)** | Färe | $(\mathbf{1}, \mathbf{1})$ | None | No |
| **Chung et al. (1997)** | Chung | $(\boldsymbol{x}, \boldsymbol{y})$ | The same input and output mixes | No |
| **Zofio et al. (2013)** | Zofio | $\dfrac{(\boldsymbol{x} - \boldsymbol{x}^*, \boldsymbol{y}^* - \boldsymbol{y})}{(\boldsymbol{P}_y \boldsymbol{y}^* - \boldsymbol{P}_x \boldsymbol{x}^*) - (\boldsymbol{P}_y \boldsymbol{y} - \boldsymbol{P}_x \boldsymbol{x})}$ | Profit maximization with exogenous price | Need |
| **This paper** | Nash | $\dfrac{(\boldsymbol{x} - \boldsymbol{x}^{N*}, \boldsymbol{y}^{N*} - \boldsymbol{y})}{(\boldsymbol{P}_y^{N*} \boldsymbol{y}^{N*} - \boldsymbol{P}_x^{N*} \boldsymbol{x}^{N*}) - (\boldsymbol{P}_y^{N} \boldsymbol{y} - \boldsymbol{P}_x^{N} \boldsymbol{x})}$ | Profit maximization with endogenous price | Need |

- where $(\boldsymbol{P}_y^{N*}, \boldsymbol{P}_x^{N*})$ is the market price vector calculated by the price functions with respect to Nash solution $(\boldsymbol{x}^{N*}, \boldsymbol{y}^{N*})$, and $(\boldsymbol{P}_y^{N}, \boldsymbol{P}_x^{N})$ is the market price vector with respect to observation $(\boldsymbol{x}, \boldsymbol{y})$.

# Nash Profit Efficiency and its Decomposition (Lee, 2016)

- Nash-profit maximization function ( $\text{NPF}^*$ ), the Nash-profit function (NPF), and the current-profit function (CPF)
  - $NPF^* = \sum_j P_j^Y\left(\bar{Y}_j^{N*}, \bar{\boldsymbol{Y}}_{(-j)}^{N*}\right) y_{rj}^{N*} - \sum_i P_i^X\left(\bar{X}_i^{N*}, \bar{\boldsymbol{X}}_{(-i)}^{N*}\right) x_{ri}^{N*} = \boldsymbol{P}_y^{N*}\boldsymbol{y}_r^{N*} - \boldsymbol{P}_x^{N*}\boldsymbol{x}_r^{N*}$
  - $NPF = \sum_j P_j^Y\left(\bar{Y}_j^{N*}, \bar{\boldsymbol{Y}}_{(-j)}^{N*}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i^{N*}, \bar{\boldsymbol{X}}_{(-i)}^{N*}\right) x_{ri} = \boldsymbol{P}_y^{N*}\boldsymbol{y}_r - \boldsymbol{P}_x^{N*}\boldsymbol{x}_r$
  - $CPF = \sum_j P_j^Y\left(\bar{Y}_j, \bar{\boldsymbol{Y}}_{(-j)}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ri} = \boldsymbol{P}_y^{N}\boldsymbol{y}_r - \boldsymbol{P}_x^{N}\boldsymbol{x}_r$

- Nash Profit Efficiency (NPE)
  - $NPE = NPF^* - CPF = (NPF^* - NPF) + (NPF - CPF)$

    $=$ efficiency in quantity change (EQC) + efficiency in price change (EPC)

Fix price, measure the difference of Nash and firm

Fix quantity, measure the effect of the price function

  - The decomposition is used to measure the change of market structure

- Nash Profit Efficiency and its Decomposition
- NPE vs. PE (profit efficiency)
  - PE must be non-negative, but NPE can be positive or negative due to the endogenous price.
  - When $\mathrm{NPE} > 0$, a firm should change its input or output mix towards a Nash solution, meaning that competition is increasing and will benefit the firm's profit.
  - $\mathrm{NPE} < 0$ implies a poor Nash solution and a firm should maintain its current competence because competition is destructive and will undermine the profit.
  - Therefore, $\mathrm{NPE}$ is a good indicator for planning and adjusting a firm's inputs and outputs according to a Nash equilibrium in an imperfectly competitive market. (CR can't)

**Proposition 1:** The DDF estimated by Nash direction $\frac{(\boldsymbol{x}-\boldsymbol{x}^{N*}, \boldsymbol{y}^{N*}-\boldsymbol{y})}{(\boldsymbol{P}_y^{N*}\boldsymbol{y}^{N*}-\boldsymbol{P}_x^{N*}\boldsymbol{x}^{N*})-(\boldsymbol{P}_y^{N}\boldsymbol{y}-\boldsymbol{P}_x^{N}\boldsymbol{x})}$ is equal to NPE. Similarly, using the direction $\frac{(\boldsymbol{x}-\boldsymbol{x}^{N*}, \boldsymbol{y}^{N*}-\boldsymbol{y})}{(\boldsymbol{P}_y^{N*}\boldsymbol{y}^{N*}-\boldsymbol{P}_x^{N*}\boldsymbol{x}^{N*})-(\boldsymbol{P}_y^{N*}\boldsymbol{y}-\boldsymbol{P}_x^{N*}\boldsymbol{x})}$ generates the DDF equal to EQC.

# Empirical Study- New York State in 1980s

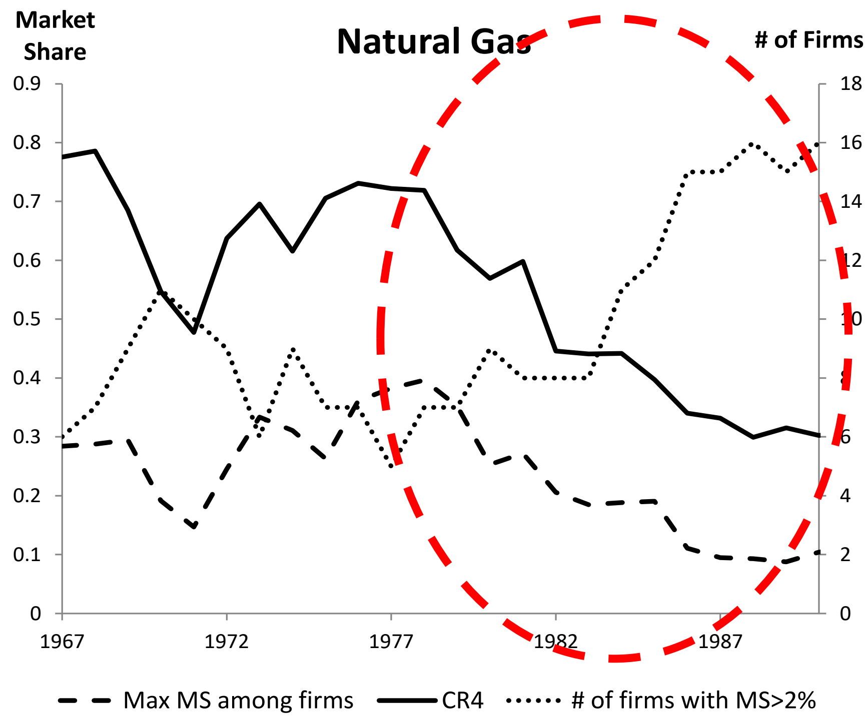
- New York State's gas and oil industry between 1981 and 1989
  - Source: New York State's Department of Environmental Conservation (DEC) (2012) and the federal government's Energy Information Administration (EIA) (2012).
  - unbalanced firm-level yearly panel data (consider 16 firms with MS > 2%)

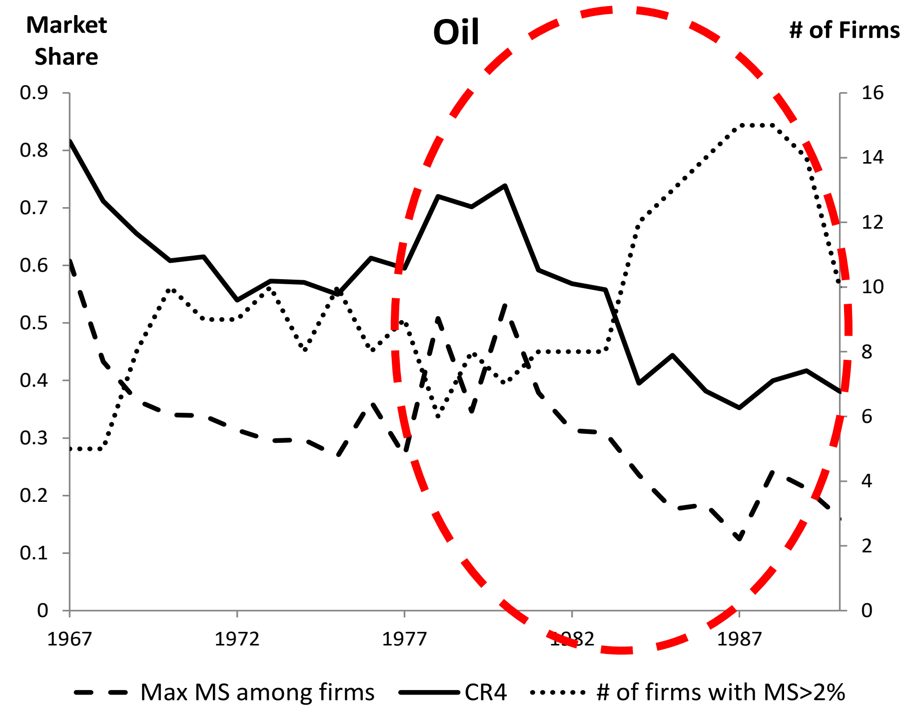
- Input (fixed input cannot be changed in the short run)
  - number of active gas wells
  - number of active oil wells
- Output
  - gas volume (unit: million cubic feet, MMcf)
  - oil volume (unit: thousand barrels, Mbbl)
- Output Price
  - the price functions for natural gas and oil significantly change before and after deregulation in 1986.

# Empirical Study- New York State in 1980s

POLab

- New York State's gas and oil industry between 1981 and 1989.




Lee (2016)

- After 1986, New York State's natural gas market became fully deregulated.
  - How to estimate the change of market structure from a productivity viewpoint?

# Empirical Study- New York State in 1980s

## Output price function

- estimate the price functions using ordinary least square (OLS) with the exception of 1986, due to the one-year transition effect after deregulation
- Oil and natural gas prices dropped in 1986 due to deregulation and an unforeseen oil glut.

- Before 1986 deregulation
  - natural gas $P_G^Y = 15000 - 0.5\bar{Y}_G - 0.01 \times 21.19 \times \bar{Y}_O$
  - oil $P_O^Y = 47968 - 21.19\bar{Y}_O - 0.01 \times 0.5 \times \bar{Y}_G$

- After 1986
- natural gas $P_G^Y = 3527 - 0.05\bar{Y}_G - 0.01 \times 0.32 \times \bar{Y}_O$
- oil $P_O^Y = 17023 - 0.32\bar{Y}_O - 0.01 \times 0.05 \times \bar{Y}_G$

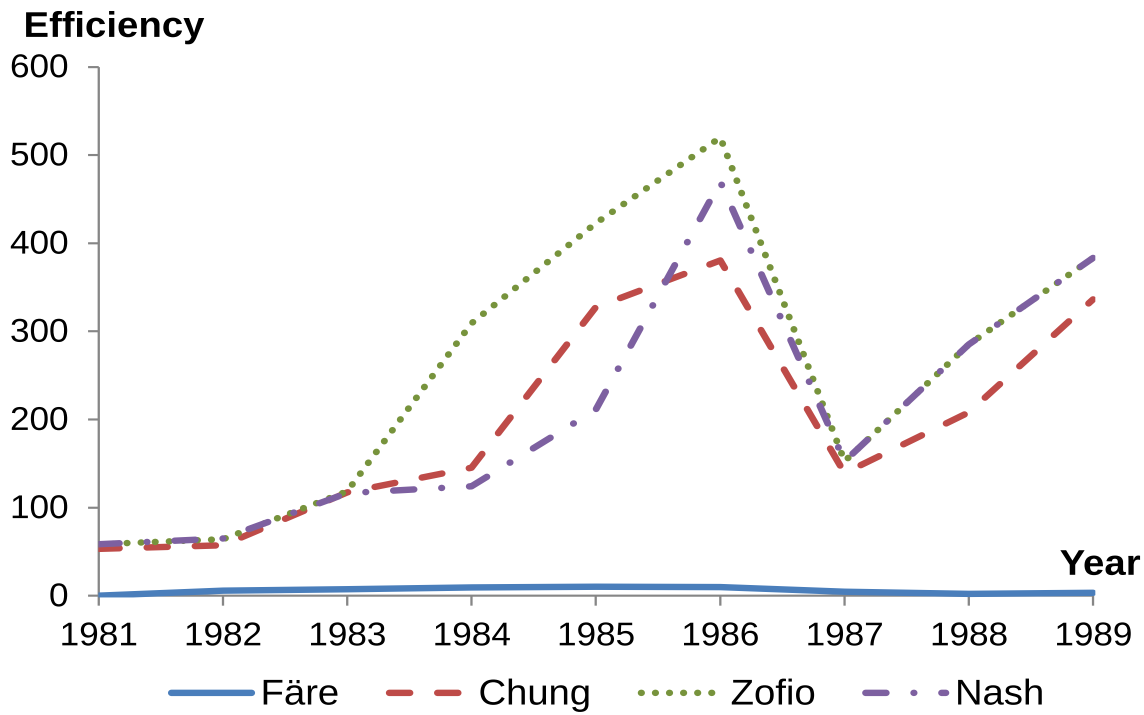
# Empirical Study- New York State in 1980s

- DDF and NPE in natural gas and oil industry, 1981–1989

| Year | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 | 1989 |
|---|---|---|---|---|---|---|---|---|---|
| Market Structure | Regulation | | | | | Deregulation | | | |
| $CR_4$ (Gas) | 0.60 | 0.45 | 0.44 | 0.44 | 0.40 | 0.34 | 0.33 | 0.30 | 0.32 |
| $CR_4$ (Oil) | 0.59 | 0.57 | 0.56 | 0.40 | 0.44 | 0.38 | 0.35 | 0.40 | 0.42 |
| DDF (Technical Inefficiency) | | | | | | | | | |
| Färe | 0.078 | 0.112 | 0.758 | 1.801 | 1.213 | 2.446 | 1.206 | 0.660 | 1.057 |
| Chung | 22.719 | 35.711 | 51.090 | 104.499 | 115.969 | 149.477 | 51.236 | 92.745 | 141.178 |
| Zofio | 2208.139 | 1529.269 | 2008.623 | 2692.962 | 3688.66 | 1784.751 | 816.905 | 658.060 | 997.326 |
| Lee | 23.770 | 5.907 | 24.712 | 29.001 | 223.376 | 50.636 | 15.247 | 57.592 | 14.600 |
| Nash | 72.764 | 133.235 | 126.806 | 85.592 | 1702.86 | 644.541 | 267.513 | 55.433 | 150.478 |
| Nash Profit Efficiency and Profit Efficiency | | | | | | | | | |
| NPE | -10864197 | -5331440 | -5073004 | -1500883 | 943950.5 | 41381.27 | -644634 | -1227897 | -1083706 |
| EQC | 22444.4 | 559499 | 963797.3 | 443484.4 | 36272.71 | 1507440 | 716997.6 | 656317.8 | 670618.2 |
| EPC | -10886641 | -5890939 | -6036802 | -1944367 | 907677.8 | -1466059 | -1361631 | -1884215 | -1754324 |
| PE | 3555284 | 2523694 | 3247856 | 8844758 | 9794941 | 3389097 | 1385309 | 1360368 | 1991247 |
| Market Preference in Efficiency | | | | | | | | | |
| MPE | -14419481 | -7855134 | -8320860 | -10345641 | -8850991 | -3347715 | -2029943 | -2588265 | -3074953 |
| EBE | -8519183 | -5283621 | -7322234 | -15116108 | -17168937 | -4749243 | -1909560 | -2779283 | -3025818 |
| EPE | 5900298 | 2571513 | 998625.7 | -4770468 | -8317946 | -1401528 | 120382.2 | -191018 | 49135.71 |

## Empirical Study- New York State in 1980s

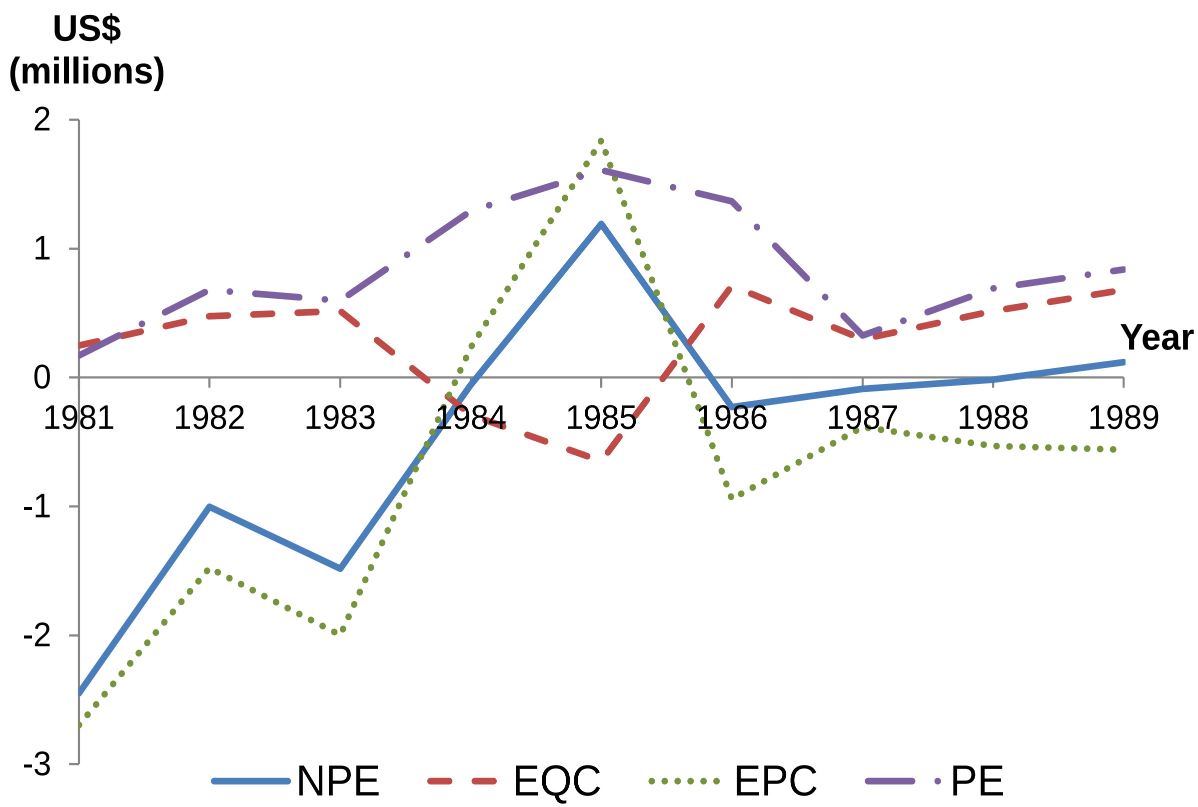
- DDF in natural gas and oil industry




- In general, increasing competitive pressure should cause firms to work harder, but the oil glut in 1986 weakens the incentive for oil and gas production and causes poor efficiency scores.
- Moreover, New York State's demand requirement is larger than its generation in the early 1980s, whereas greater imports of natural gas after 1986 lead to greater competition in 1987 and DDF scores drop.

# Empirical Study- New York State in 1980s

- The result of NPE in 1981–1989 is consistent with the change in CR, but NPE provides a more detailed guideline for driving productivity, and integrate 2 CRs (oil and gas) into one NPE index.
- Typical PE presents only positive values and poorly distinguishes the insights from similar efficiency levels between 1981–1983 and 1987–1989.
- Moreover, the change of NPE mainly derives from the price effect, i.e., EPC, rather than EQC.



# Nash-Profit Efficiency for Market Structure

## Contribution

- alternative measure of the economic efficiency in an imperfectly competitive market
- Consider the endogenous price
- Provide Nash direction used in DDF for efficiency estimation
- an empirical study of the oil and natural gas industry in New York State 1981-1989
- The proposed NPE measure, which complements the typical PE and CR, can capture the change of market structure from the perspective of productivity analysis.

## Further Research

- revised for undesirable outputs in imperfectly competitive markets
- Chambers et al. (2014) explicitly decomposed the Lerner index into the three components: cost elasticity, Farrell output measure of technical efficiency, and Georgescu-Roegen return to the dollar. NPE could be linked to the typical market power indices assessed by price cost margin (PCM) and CR.

European Journal of Operational Research 255 (2016) 659–663

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Short Communication

# Nash-profit efficiency: A measure of changes in market structures

CrossMark

Chia-Yen Lee*

*Institute of Manufacturing Information and Systems, National Cheng Kung University, Tainan City 701, Taiwan*

A R T I C L E I N F O

*Article history:*
Received 20 October 2015
Accepted 26 May 2016
Available online 3 June 2016

*Keywords:*
Data envelopment analysis
Nash equilibrium
Imperfectly competitive market
Mixed complementarity problem
Oil and natural gas industry

A B S T R A C T

Imperfectly competitive markets can be characterized by endogenous prices, limited or no competition, and the exercise of market power. To address the resulting dysfunctionality, this study proposes an alternative efficiency measure estimated by the directional distance function (DDF) with the direction toward Nash equilibrium, and develops the Nash-profit efficiency (NPE) and its decomposition which complements the typical profit efficiency measure. We model the production possibility set and the price functions of inputs and outputs, and then develop the mixed complementarity problem (MiCP). We validate the model with an empirical study of the oil and natural gas industry in New York State between 1981 and 1989. The results show that before 1984, firms exploited a less competitive market; that between 1984 and 1986, the number of new entrants transformed the market; and that after 1986, no firms could exercise market power due to market restructuring (deregulation) and an unforeseen oil glut. Based on the results, we conclude that the direction toward Nash equilibrium can be justified for efficiency estimation in imperfectly competitive markets, and that NPE is appropriate for investigating changes in market structures.

# Mixed-Strategy Nash Equilibrium

Lee, Chia-Yen, 2018. Mixed-Strategy Nash Equilibrium in Data Envelopment Analysis. European Journal of Operational Research, 266 (3), 1013-1024.

# Outline

- Introduction
- Mixed-Strategy Nash Equilibrium
- Efficiency Measurement via Mixed Strategy
- Direction Measurement via the Mixed Strategy
- Empirical Study
- Conclusions

# Introduction

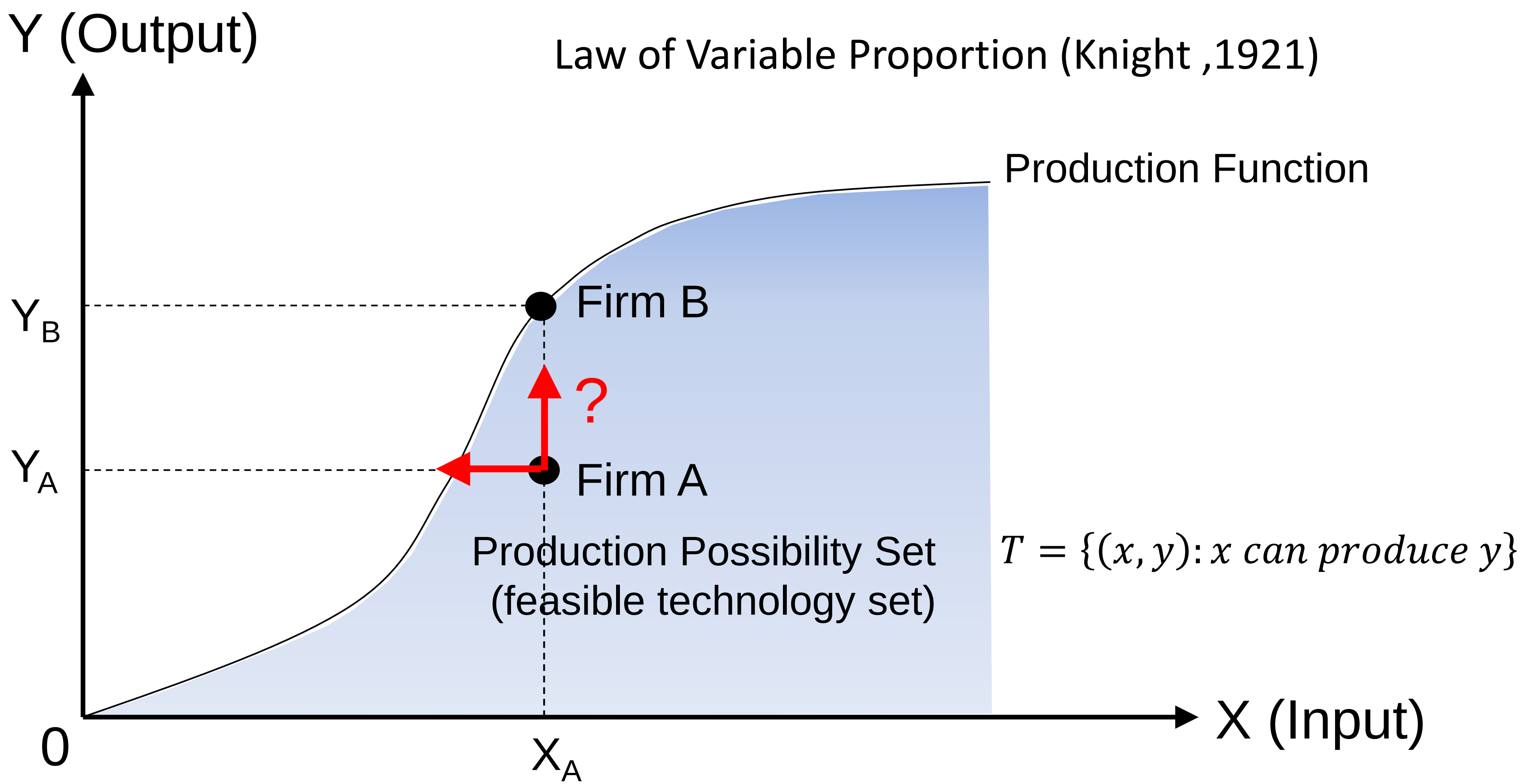
- Energy markets are imperfectly competitive
  - Data Envelopment Analysis (DEA) implicitly assumes an exogenous price in a perfectly competitive market (Cherchye, Kuosmanen, & Post, 2002)
  - In an imperfectly competitive market, firms exercising affect the market price (i.e., price is endogenous based on demand function).
    - non-cooperative game (Nash, 1951) (Lee & Johnson, 2015)
    - a firm may overestimate the revenue when expanding output by assuming exogenous price (Lee, 2016).

## DEA Games

- The seminal works of the unconstrained and constrained two-person zero-sum games were developed by Banker (Banker, 1980)
- Aparico et al. (J. Aparicio, Landete, Monge, & Sirvent, 2008) proposed the iterative multi-unit combinatorial auctions based on a linear anonymous pricing scheme. They emphasized on determining the price of any bundle of items through a computational experiment.
- Wu et al. (Wu, Liang, Yang, & Yan, 2009) proposed the Nash bargaining game to improve the non-uniqueness and average properties of the cross-efficiency measure.
- Lozano (Lozano, 2013) extended the linear production game to a general production function and formulated a DEA production collaborative game. The full-cooperation scenario and the partial-cooperation scenario are investigated
- However, some unsolved issues remain in the literature.
  - DEA games assumes perfectly competitive market
  - Orientation is not fixed in a dynamic and uncertain environment

# Introduction

- Multi-oriented efficiency measure?




A production function is a function that represents "maximum outputs" that can be achieved using input vector $\mathbb{x}$.

# Introduction

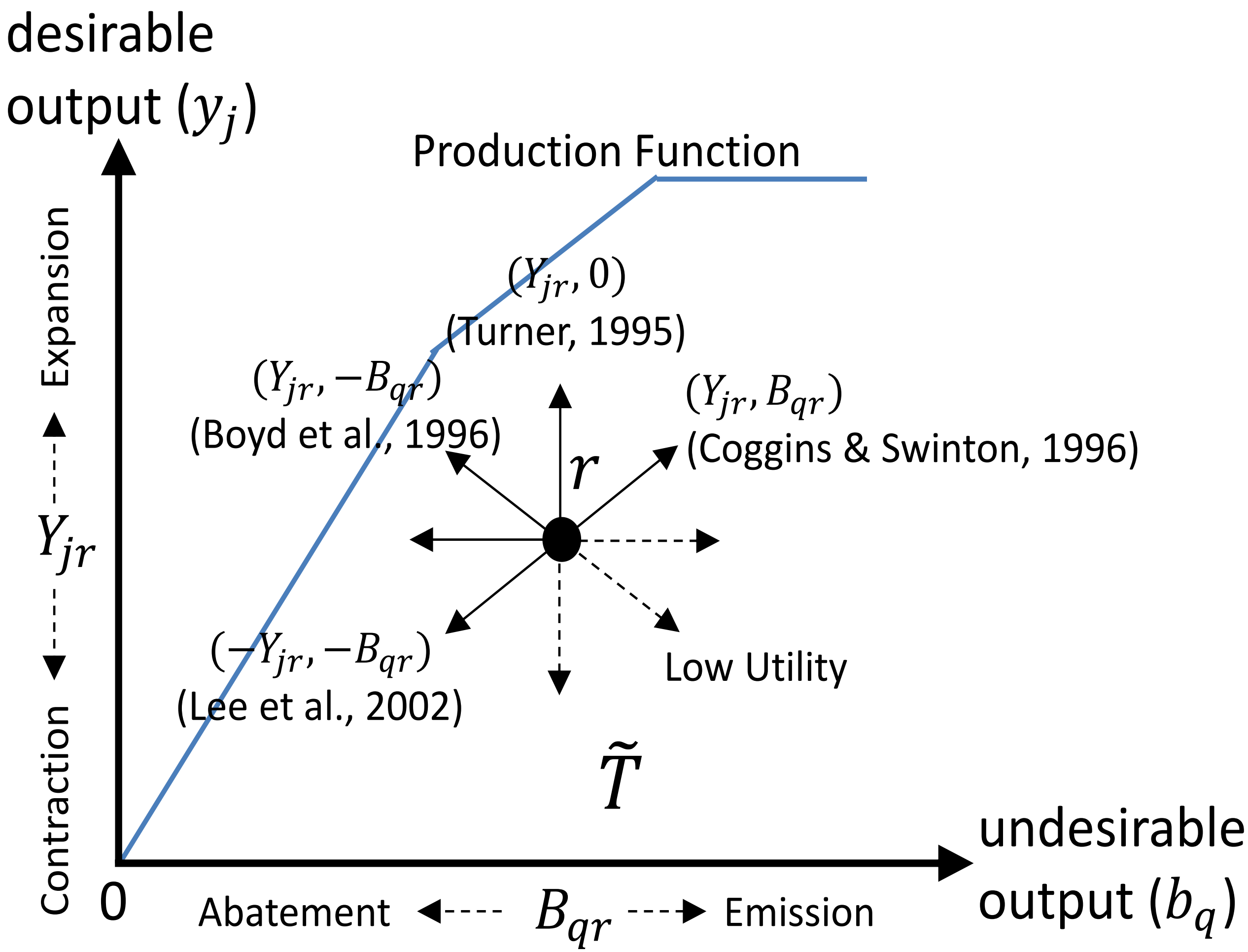
- Mixed-strategy measure (MSM)
  - A mixed strategy assigns a probability to each pure strategy, allowing a player to randomly select a pure strategy.
  - an MSM is a rational strategy for addressing the uncertainty from other players.
  - A strategy set is the set of pure strategies available to a player; each pure strategy should be non-dominated or have the same utility/payoff.
  - There is no mixed strategy if one pure strategy dominates all other pure strategies in a strategy set.
  - This study extends the Nash efficiency measure (Lee & Johnson, 2015) to the MSM.
- Environmental regulation
  - considering the environmental regulation in an energy market
  - strategic consistency versus environmental consistency
- Empirical study of China's electric power industry in 2010

- "Weak Disposability (Fare, Grosskopf, & Pasurkajr, 2007):
  - Free (or strong) disposability of desirable outputs
    Given $(\boldsymbol{x}, y, \boldsymbol{b}) \in T$ and $0 \leq y' \leq y$, then $(\boldsymbol{x}, y', \boldsymbol{b}) \in T$. (1.1)
  - Weak disposability of desirable outputs and undesirable outputs
    Given $(\boldsymbol{x}, y, \boldsymbol{b}) \in T$ and $0 \leq \rho \leq 1$, then $(\boldsymbol{x}, \rho y, \rho \boldsymbol{b}) \in T$. (1.2)
  - Nulljointness of desirable outputs and undesirable outputs
    Given $(\boldsymbol{x}, y, \boldsymbol{b}) \in T$ and $\boldsymbol{b} = 0$, then $y = 0$. (1.3)

- Podinovski's convex technology $T$ with cap constraint
  - $$\tilde{T} = \left\{ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \middle| \begin{array}{c} \sum_{k \in K} (\lambda_k + \mu_k) X_{ik} \leq x_i, \forall i \in I; \\ \sum_{k \in K} \lambda_k Y_{jk} \geq y_j;\ \forall j \in J; \\ \sum_{k \in K} \lambda_k B_{qk} \leq b_q, \forall q \in Q; \\ \sum_{k \in K} (\lambda_k + \mu_k) = 1; \\ \hat{B}_q \leq \hat{B}_q^{CAP}, \forall q \in Q; \\ \lambda_k, \mu_k \geq 0, \forall k \in K \end{array} \right\} \quad (2)$$

# Mixed-Strategy Nash Equilibrium

- Pure Strategies (i.e. Direction Vector)



# Mixed-Strategy Nash Equilibrium

## Nash profit function (NPF)

- $NPF_r^{DEA*} = \max_{y_r, b_{qr}, x_{ir}} \sum_{j\in J} P_j^y(\hat{Y}) y_{jr} - \sum_{i\in I} P_i^x(\hat{X}) x_{ir} - \sum_{q\in Q} P_q^b(\hat{B}) b_{qr}$ (3.1)
- $\sum_{k\in K}(\lambda_{kr} + \mu_{kr}) X_{ik} \le x_{ir}, \forall i \in I;$ (3.2)
- $\sum_{k\in K} \lambda_{kr} Y_{jk} \ge y_{jr}, \forall j \in J;$ (3.3)
- $\sum_{k\in K} \lambda_{kr} B_{qk} \le b_{qr}, \forall q \in Q;$ (3.4)
- $\sum_{k\in K}(\lambda_{kr} + \mu_{kr}) = 1;$ (3.5)
- $\hat{B}_q \le \hat{B}_q^{CAP}, \forall q \in Q;$ (3.6)
- $\sum_{s\in S} \delta_{sr} = 1;$ (3.7)
- $x_{ir} = X_{ir} + \sum_{s\in S} \delta_{sr} g_{sir}^x, \forall i \in I;$ (3.8)
- $y_{jr} = Y_{jr} + \sum_{s\in S} \delta_{sr} g_{sjr}^y, \forall j \in J;$ (3.9)
- $b_{qr} = B_{qr} + \sum_{s\in S} \delta_{sr} g_{sqr}^b, \forall q \in Q;$ (3.10)
- $x_{ir}, y_{jr}, b_{qr}, \lambda_{kr}, \mu_{kr}, \delta_{sr} \ge 0, \forall i \in I, j \in J, q \in Q, k \in K, s \in S$ (3.11)

## Mixed Complementarity Problem (MiCP)

- $0 \leq \delta_{sr} \perp \sum_{j \in J} \left(P_j^{y_0} - \kappa_j^y \hat{Y}_j\right) g_{sjr}^y - \sum_{j \in J} \left(\kappa_j^y g_{sjr}^y\right) \left(Y_{jr} + \sum_{s \in S} \delta_{sr} g_{sjr}^y\right)$
  $- \sum_{i \in I} \left(P_i^{x_0} + \kappa_i^x \hat{X}_i\right) g_{sir}^x - \sum_{i \in I} (\kappa_i^x g_{sir}^x) \left(X_{ir} + \sum_{s \in S} \delta_{sr} g_{sir}^x\right) - \sum_{q \in Q} \left(P_q^{b_0} + \right.$

# Efficiency Measurement via Mixed Strategy

## MSM by Directional Distance Function (DDF)

- Given the direction vector $\boldsymbol{g} = \left(\boldsymbol{g}^x, \boldsymbol{g}^y, \boldsymbol{g}^b\right)$, where $\boldsymbol{g}^x \in \mathbb{R}_+^{|I|}$, $\boldsymbol{g}^y \in \mathbb{R}_+^{|J|}$, and $\boldsymbol{g}^b \in \mathbb{R}_+^{|Q|}$, and letting $\gamma$ be the decision variable representing the estimate of the technical efficiency

- DDF: $\vec{D}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g}^x, \boldsymbol{g}^y, \boldsymbol{g}^b) = \max\{\gamma | (\boldsymbol{x} + \gamma \boldsymbol{g}^x, \boldsymbol{y} + \gamma \boldsymbol{g}^y, \boldsymbol{b} + \gamma \boldsymbol{g}^b) \in \tilde{T}\}$
  - If the optimal solution $\gamma^*$ is zero, the firm is efficient; otherwise it is inefficient.

- MSM: Set $\left(\boldsymbol{g}_{sr}^x, \boldsymbol{g}_{sr}^y, \boldsymbol{g}_{sr}^b\right) = (g_{s1r}^x, \dots, g_{s|I|r}^x, g_{s1r}^y, \dots, g_{s|J|r}^y, g_{s1r}^b, \dots, g_{s|Q|r}^b\boldsymbol{)}$ for one specific direction $s$ with $\delta_s^* > 0$ obtained from MiCP.
  - $\gamma_s^* = \vec{D}_s(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}; \boldsymbol{g}_s^x, \boldsymbol{g}_s^y, \boldsymbol{g}_s^b) = \max\{\gamma_s | (\boldsymbol{x} + \gamma_s \boldsymbol{g}_s^x, \boldsymbol{y} + \gamma_s \boldsymbol{g}_s^y, \boldsymbol{b} + \gamma_s \boldsymbol{g}_s^b) \in \tilde{T}\}$

- Therefore, the MSM is defined as $\vec{D}_r^{MSM} = \sum_{s \in S} \delta_{sr}^* \gamma_{sr}^*$.

## Environmental consistency (EC)

- EC captures the difference between the optimal solutions generated by MiCP (4) with a cap constraint and the solutions from MiCP (4) without a cap constraint.

$$EC = 1 - \frac{1}{2}\sqrt{\sum_{i\in I}\left(\bar{g}_{ir}^{xE} - \bar{g}_{ir}^{xW}\right)^2 + \sum_{j\in J}\left(\bar{g}_{jr}^{yE} - \bar{g}_{jr}^{yW}\right)^2 + \sum_{j\in J}\left(\bar{g}_{qr}^{bE} - \bar{g}_{qr}^{bW}\right)^2}$$

- If $EC = 1$, then it implies that the environment policy is not opposed to the economic growth when the firm pursues the latter in a competitive environment.

# Direction Measurement via the Mixed Strategy

## Strategic consistency (SC)

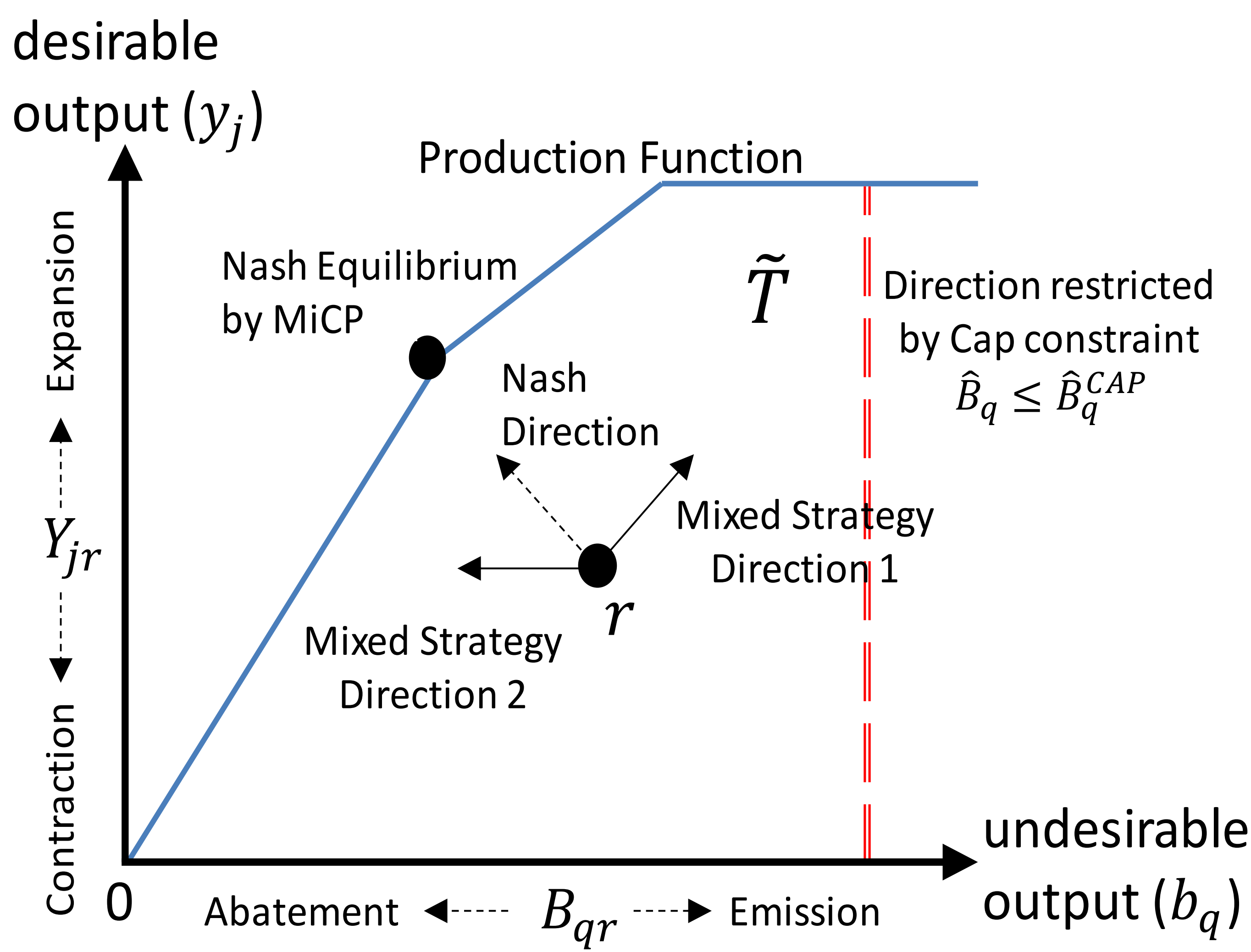
- SC is measured by the difference between the direction generated by MiCP (4) with a cap constraint and the Nash direction without mixed strategy.

$$SC = \begin{cases} 1-\frac{1}{2}\sum_{s\in S}\delta_{sr}^{*E}\sqrt{\sum_{i\in I}\left(g_{sir}^{x}-g_{ir}^{xN}\right)^2+\sum_{j\in J}\left(g_{sjr}^{y}-g_{jr}^{yN}\right)^2+\sum_{q\in Q}\left(g_{sq}^{b}-g_{qr}^{bN}\right)^2}\text{, if firm } r \text{ takes ac} \\ 1-\frac{1}{2}\sqrt{\sum_{i\in I}\left(g_{ir}^{xN}\right)^2+\sum_{j\in J}\left(g_{jr}^{yN}\right)^2+\sum_{q\in Q}\left(g_{qr}^{bN}\right)^2}=0.5\text{, if firm } r \text{ takes "D/N"} \end{cases}$$

- If $SC = 1$, then it implies that the short-term strategy matches the long-term goal when the firm pursues the profit maximization under a competitive environment.

# Direction Measurement via the Mixed Strategy

- Nash direction v.s. mixed strategy directions



# Empirical Study

- China's regional electric power industry sectors in 2010
- 30 provincial-level regions
- Inputs, Outputs and Prices
  - One desirable output: the annual amount of electricity generated in Megawatt-hours (MWh).
    - The electricity price $P^Y(\hat{Y}) := 55{,}000{,}000 - 330.77\hat{Y}$.
  - Three undesirable outputs: the annual amount in tons of $CO_2$, $SO_2$ and $NO_x$.
    - The costs of $CO_2$, $SO_2$, and $NO_x$ per ton are RMB$ 36.72, 701.1, and 6698.3, respectively according to the studies Lee and Zhou (2015)
  - Three input: nameplate capacity, labor force, and energy consumption.
    - the price of nameplate capacity is RMB$ 250,000 per MW per year
    - the average cost of labor assumes RMB$ 4500 per month
    - the price of coal is RMB$ 590 per tons

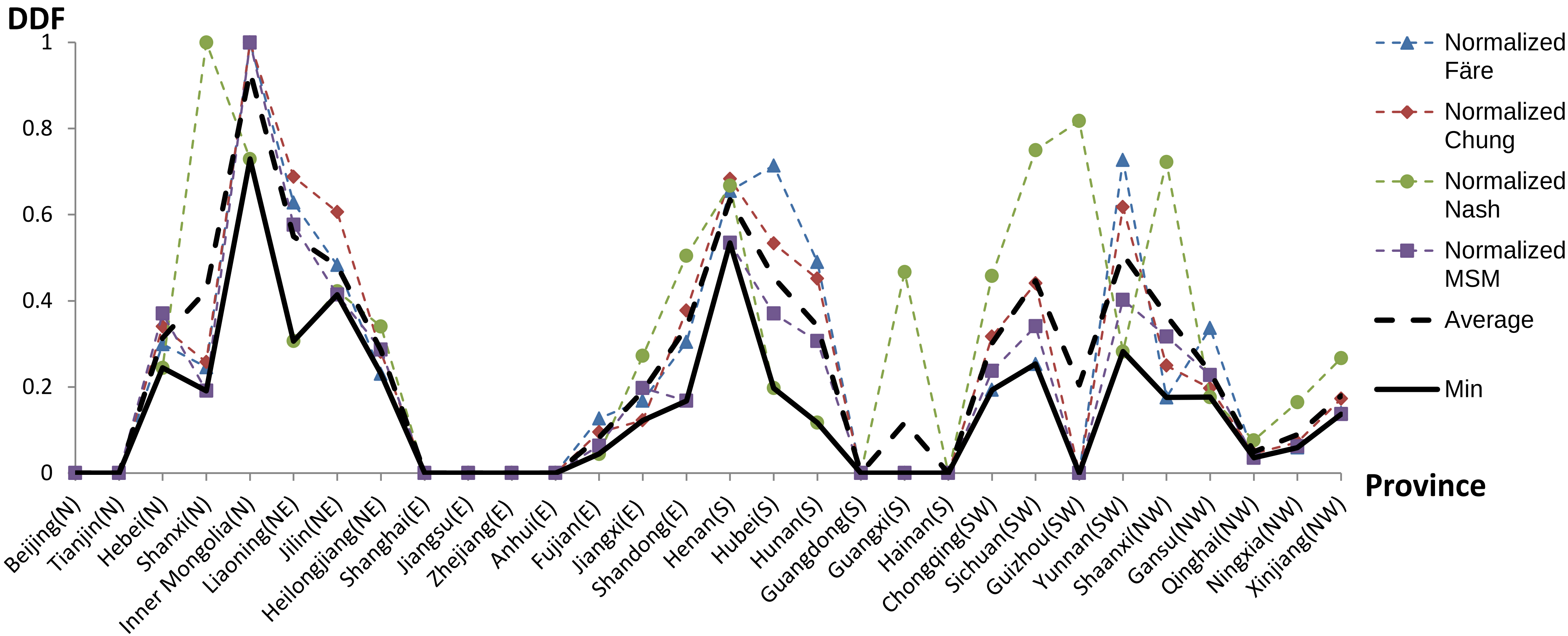
# Empirical Study

- Three potential direction vectors $(-Y_{jr}, -B_{qr})$ , $(0, -B_{qr})$ , and $(Y_{jr}, B_{qr})$ as scenario $s_1$, $s_2$, and $s_3$
- Excellent performance: Beijing, Tianjin, Shanghai, Jiangsu, Zhejiang, Anhui, Guangdong, and Hainan.
- Poor performance: Inner Mongolia
- Most provinces employ the do-nothing (if located on the efficient frontier) or the pure strategy (only one among three strategies equal to one).
- Inner Mongolia, Heilongjiang, and Jiangxi provinces use the mixed strategy, they are inefficient in almost five measures.

| No. | Province | Area | Population (unit: $10^3$) | Nameplate Capacity (MW) | Färe | Chung | Nash | EV | MSM | MSM solution $(\delta^*_{s_1}, \delta^*_{s_2}, \delta^*_{s_3})$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Beijing | N | 19,612 | 6310 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 2 | Tianjin | N | 12,938 | 10940 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 3 | Hebei | N | 71,854 | 42150 | 416 | 60795 | 88113 | 3047 | 149119 | (0,1,0) |
| 4 | Shanxi | N | 35,712 | 44290 | 342 | 46055 | 360795* | 76904* | 76904 | (0,0,1) |
| 5 | Inner Mongolia | N | 24,706 | 63720 | 1390* | 178706* | 263119* | 341259* | 402539* | (0,0.88,0.12) |
| 6 | Liaoning | NE | 43,746 | 32280 | 873* | 122990* | 110597 | 8484 | 232060* | (0,1,0) |
| 7 | Jilin | NE | 27,462 | 20350 | 671 | 108343* | 152382 | 12339 | 166784* | (0,1,0) |
| 8 | Heilongjiang | NE | 38,312 | 19650 | 320 | 50226 | 122871 | 66900 | 115448 | (0,0.28,0.72) |
| 9 | Shanghai | E | 23,019 | 18580 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 10 | Jiangsu | E | 78,660 | 64700 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 11 | Zhejiang | E | 54,427 | 57210 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 12 | Anhui | E | 64,501 | 29330 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 13 | Fujian | E | 36,894 | 34050 | 176 | 16933 | 16040 | 25674 | 25674 | (0,0,1) |
| 14 | Jiangxi | E | 44,567 | 16320 | 234 | 21889 | 98133 | 74768 | 79537 | (0,0.07,0.93) |
| 15 | Shandong | E | 95,793 | 62680 | 423 | 67484 | 182052 | 67484 | 67484 | (0,0,1) |
| 16 | Henan | S | 94,024 | 50570 | 910* | 122084* | 240639 | 215113* | 215113* | (0,0,1) |
| 17 | Hubei | S | 57,238 | 49060 | 993* | 95303 | 71118 | 9827 | 149057 | (0,1,0) |
| 18 | Hunan | S | 65,684 | 29900 | 681 | 80645 | 42214 | 123305* | 123305 | (0,0,1) |
| 19 | Guangdong | S | 104,303 | 70890 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 20 | Guangxi | S | 46,027 | 25150 | 0 | 0 | 168352 | 0 | 0 | (0,0,0) |
| 21 | Hainan | S | 8,672 | 3860 | 0 | 0 | 0 | 0 | 0 | (0,0,0) |
| 22 | Chongqing | SW | 28,846 | 11550 | 269 | 56691 | 165181 | 5105 | 95374 | (0,1,0) |
| 23 | Sichuan | SW | 80,418 | 42240 | 352 | 78737 | 270582* | 7543 | 137258 | (0,1,0) |
| 24 | Guizhou | SW | 34,746 | 32840 | 0 | 0 | 294978* | 0 | 0 | (0,0,0) |
| 25 | Yunnan | SW | 45,966 | 36160 | 1010* | 110376* | 101773 | 13166 | 161986* | (0,1,0) |
| 26 | Shaanxi | NW | 37,327 | 23580 | 244 | 44575 | 260654* | 3306 | 127615 | (0,1,0) |
| 27 | Gansu | NW | 25,575 | 21550 | 468 | 35174 | 63559 | 91626* | 91626 | (0,0,1) |
| 28 | Qinghai | NW | 5,627 | 12620 | 55 | 8141 | 27473 | 886 | 14309 | (0,1,0) |
| 29 | Ningxia | NW | 6,301 | 12910 | 82 | 12760 | 59211 | 24862 | 24862 | (0,0,1) |
| 30 | Xinjiang | NW | 21,813 | 16070 | 195 | 30816 | 96255 | 55123 | 55123 | (0,0,1) |
| | No. of Eff. Prov. | | | | 10 | 10 | 8 | 10 | 10 | |

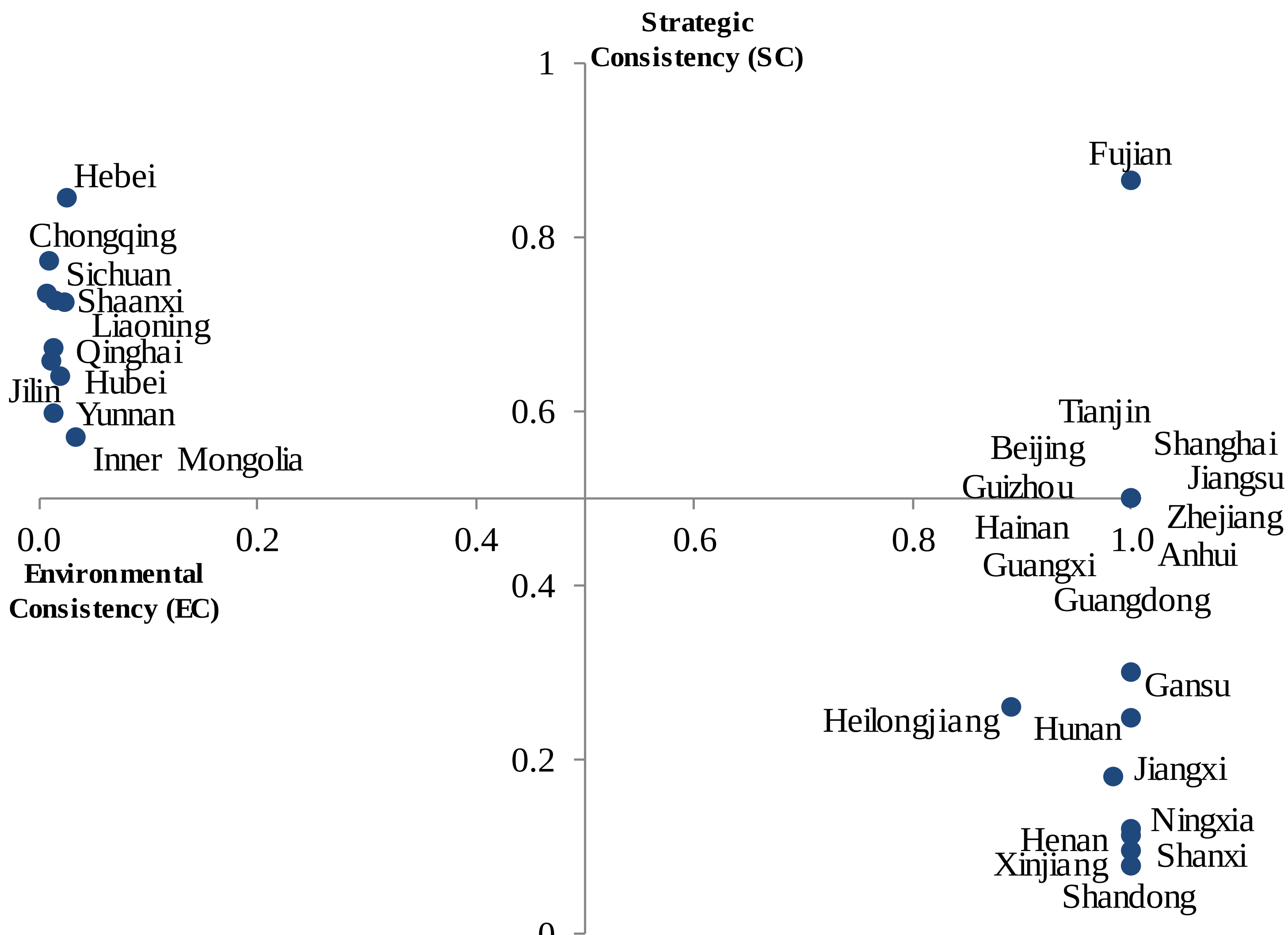
# Empirical Study

- Efficiency sorted by geographic area
  - East (E) shows better while North (N) and Northeast (NE) show poor.
  - The direction used in the Nash method differs significantly from the other three measures used in Shanxi, Guangxi, and Guizhou provinces (i.e., it implies a potential inefficiency in the three provinces).



# Empirical Study

## Environmental Consistency and Strategic Consistency



# Conclusion

## Summary

- Propose MSM
- We found that the probabilistic multi-oriented efficiency measure was justified when addressing market changes and uncertain competition.
- Focusing on "efficiency score" and investigating "direction" provided managerial insights into supporting business strategy development.

## Empirical Study

- An empirical study of China's electric power industry in 2010 validated the proposed MSM and offered guidelines for driving productivity.
  - Fare and Chung methods: are optimistic using an ad hoc direction in the energy market without considering endogenous price
  - Nash method: is pessimistic because its "only one and ideal" benchmark representing a long-run steady state restricts the adjustable flexibility of the product-mix and underestimates the efficiency score.
- the more flexible MSM method generating province-specific multiple benchmarks suggests a direction in the short run.

# Mixed-Strategy Nash Equilibrium

European Journal of Operational Research 266 (2018) 1013–1024

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

## Mixed-strategy Nash equilibrium in data envelopment analysis

Chia-Yen Lee

*Institute of Manufacturing Information and Systems, National Cheng Kung University, Tainan City 701, Taiwan*

A R T I C L E   I N F O

*Article history:*
Received 18 February 2017
Accepted 23 October 2017
Available online 31 October 2017

*Keywords:*
Data envelopment analysis
Mixed-strategy measure
Nash equilibrium
Mixed complementarity problem
China's electric power industry

A B S T R A C T

In a typical productivity analysis, the efficiency measure assumes a perfectly competitive market with an endogenous price and depends on a fixed orientation with respect to a specific firm, such as an input-oriented measure. When an imperfectly competitive market affects the endogenous price, however, firms may take a "mixed strategy" approach to address uncertain competition. This study proposes a Mixed Strategy Measure (MSM). We construct a model embedded with a data envelopment analysis (DEA) framework, that identifies the mixed-strategy Nash equilibrium in the first stage and considers a probabilistic and multi-oriented efficiency measure in the second stage. Based on the environmental regulation and a typical Nash measure, we create two indices – environmental consistency and strategic consistency – to support the business roadmap development. We validate the proposed MSM with an empirical study of China's electric power industry. The results find that the proposed MSM complements the Nash measure, and the MSM model successfully supports our two managerial strategies: invest in capacity expansion or develop emission abatement technology.

# Nash Marginal Abatement Costs

Lee, Chia-Yen, and Ke Wang, 2019. Nash Marginal Abatement Cost Estimation of Air Pollutant Emissions Using the Stochastic Semi-Nonparametric Frontier. European Journal of Operational Research, 273 (1), 390-400.

# How much does it cost to abate one extra unit of $CO_2$?

The shadow prices (SP) of pollutants are used as a reference value to the allowance price in the trading market (Lee et al., 2002).

# Literatures for MAC estimation

- Profit Maximization

Good Output   Bad Output   Input

$$\pi(p_y, p_b, p_x) = \max_{y,b,x} p_y' y - p_b' b - p_x' x$$

s.t. $F(x, y, b) = 0$ (Production Transformation Function)

- Lagrange function: $\max_{y,b,x} p_y' y - p_b' b - p_x' x + \varphi F(x, y, b)$
- First-order conditions (FOCs):
  - $p_{y_j} + \varphi \frac{\partial F(x,y,b)}{\partial y_j} = 0$
  - $-p_{b_q} + \varphi \frac{\partial F(x,y,b)}{\partial b_q} = 0$
  - $-p_{x_i} + \varphi \frac{\partial F(x,y,b)}{\partial x_i} = 0$
  - $F(x, y, b) = 0$

- Marginal Abatement Cost

$$p_{b_q} = p_{y_j} \left( \frac{\partial F(x,y,b)}{\partial b_q} \Big/ \frac{\partial F(x,y,b)}{\partial y_j} \right)$$

How to calculate the derivative of a production function?

- Stochastic Frontier Analysis (SFA) (Färe et al., 2005)
  - Parametric method
  - Translog functional form
  - Directional distance function

$$\begin{aligned}
\ln D(x,y,b) = {} & \alpha_0 + \sum_i \alpha_i \ln x_i + \sum_j \alpha_j \ln y_j + \sum_k \alpha_k \ln b_k \\
& + \frac{1}{2}\sum_i \sum_{i'} \gamma_{ii'} \ln x_i \ln x_{i'} + \frac{1}{2}\sum_j \sum_{j'} \gamma_{jj'} \ln y_j \ln y_{j'} \\
& + \frac{1}{2}\sum_k \sum_{k'} \gamma_{kk'} \ln b_k \ln b_{k'} + \sum_j \sum_k \gamma_{jk} \ln y_j \ln b_k \\
& + \sum_i \sum_j \beta_{ij} \ln x_i \ln y_j + \sum_i \sum_k \beta_{ik} \ln x_i \ln b_k \gamma_{ii'} \\
& = \gamma_{i'i},\ i \neq i';\ \gamma_{jj'} = \gamma_{j'j},\ j \neq j';\gamma_{kk'} = \gamma_{k'k}, k \neq k
\end{aligned}$$

$$\begin{aligned}
\text{Min} & \sum_n \left[\vec{D}_o(x^n, y^n, b^n; g_y, -g_b) - 0\right] \\
\text{s.t.} \ & \vec{D}_o(x^n, y^n, b^n; g_y, -g_b) \geqslant 0; \\
& \partial\vec{D}_o(x^n, y^n, b^n; g_y, -g_b)/\partial y^n \leqslant 0; \\
& \partial\vec{D}_o(x^n, y^n, b^n; g_y, -g_b)/\partial b^n \geqslant 0; \\
& \partial\vec{D}_o(x^n, y^n, b^n; g_y, -g_b)/\partial x^n \geqslant 0; \\
& g_y \sum_j \alpha_j - g_b \sum_k \alpha_k = -1; \\
& g_y \sum_j \sum_{j'} \gamma_{jj'} - g_b \sum_j \sum_k \gamma_{jk} = 0; \\
& g_y \sum_j \sum_k \gamma_{jk} - g_b \sum_k \sum_{k'} \gamma_{kk'} = 0; \\
& g_y \sum_i \sum_j \beta_{ij} - g_b \sum_i \sum_k \beta_{ik} = 0; \\
& g_y^2 \sum_j \sum_{j'} \gamma_{jj'} + g_b^2 \sum_k \sum_{k'} \gamma_{kk'} - g_y g_b \sum_j \sum_k \gamma_{jk} = 0; \\
& \gamma_{ii'} = \gamma_{i'i},\ i \neq i';\ \gamma_{jj'} = \gamma_{j'j},\ j \neq j',\ \gamma_{kk'} = \gamma_{k'k},\ k \neq k'
\end{aligned}$$

# Literatures for MAC estimation

- Data Envelopment Analysis (Lee et al. 2002)
  - Nonparametric method
  - Directional distance function
  - Dual variables

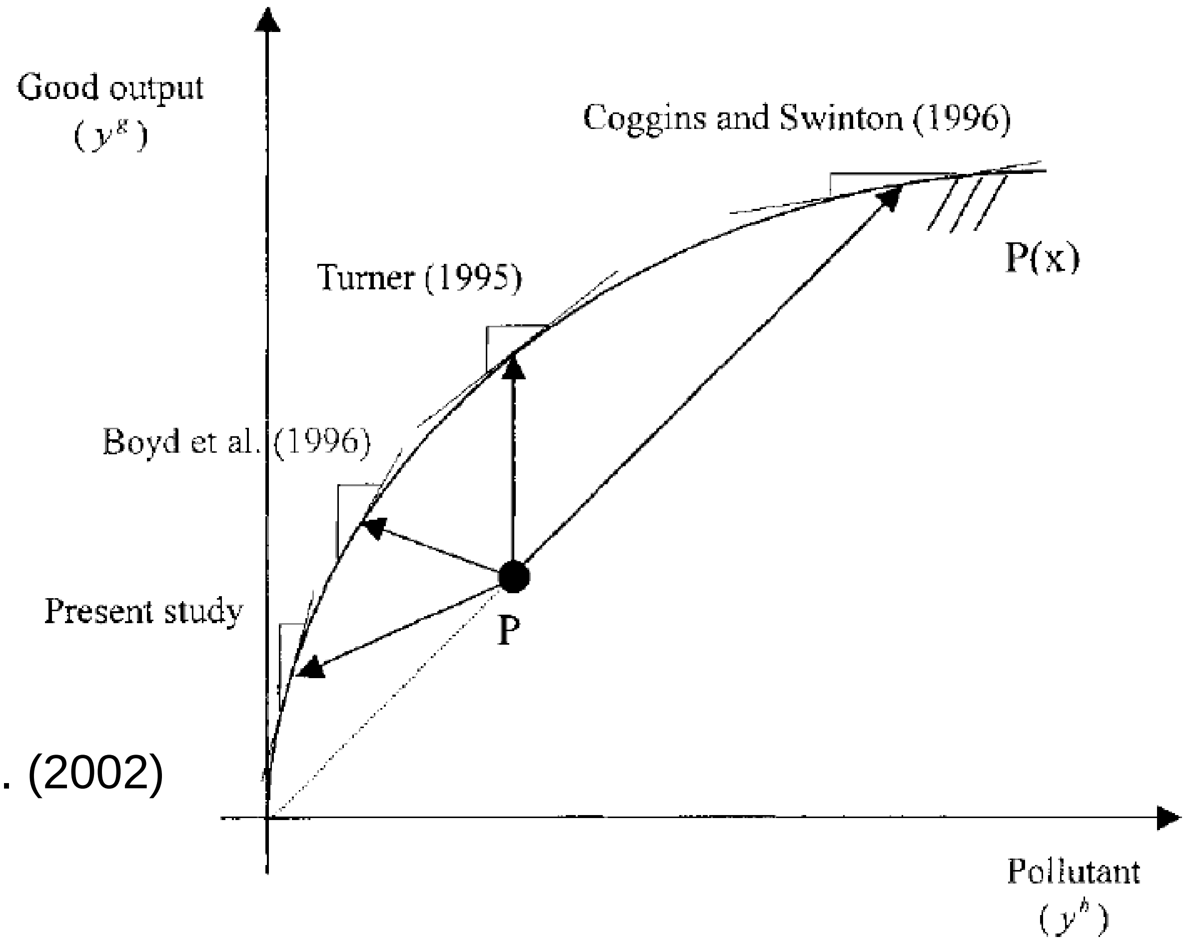
$$
\begin{aligned}
\vec{D}_o(x, y, b; g_y, g_b) &= \max_{\lambda\beta} \beta \\
\text{s.t. } Y\lambda &\geqslant (1 + \beta g_y) y^n; \\
B\lambda &= (1 - \beta g_b) b^n; \\
X\lambda &\leqslant x^n; \\
\beta, \lambda &\geqslant 0
\end{aligned}
$$

However…

There are some issues in previous studies when estimating MAC.

# Remaining Issues in literatures…

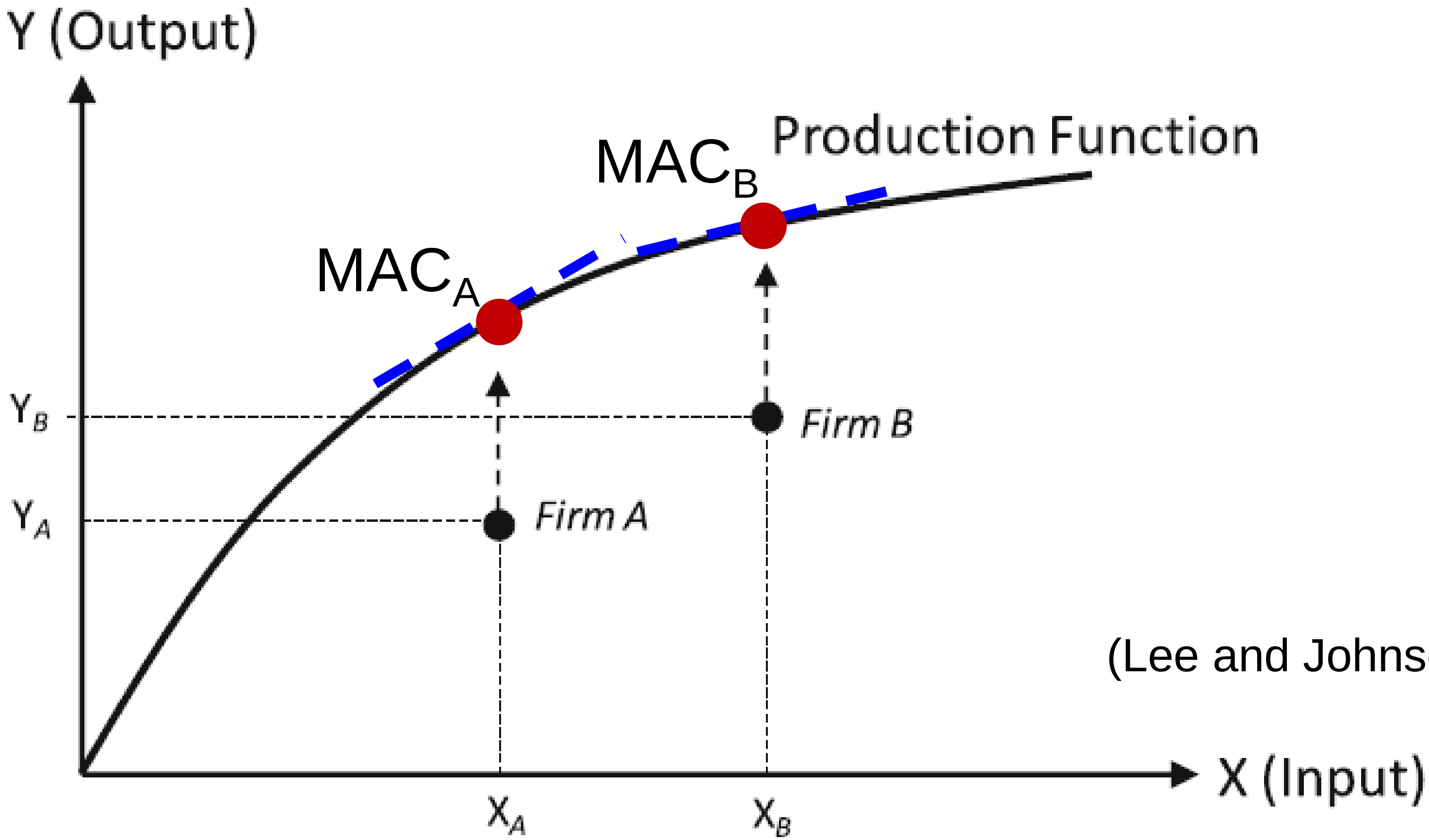
- Issue 1: direction used for projecting inefficient firms to frontier will significantly affect the MAC estimation (Lee et al. 2002; Zhou et al. 2014)




Lee et al. (2002)

# Remaining Issues in literatures…

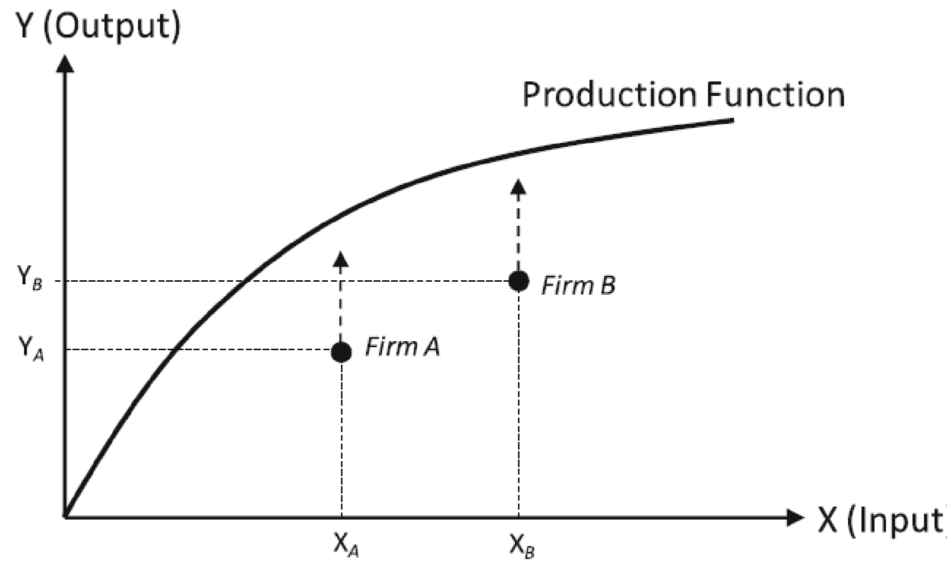
- Issue 2: average or weighted average of multiple firm-specific MACs by projecting the multiple inefficient firms to the frontier. Average may bias the MAC if the outlier exists.

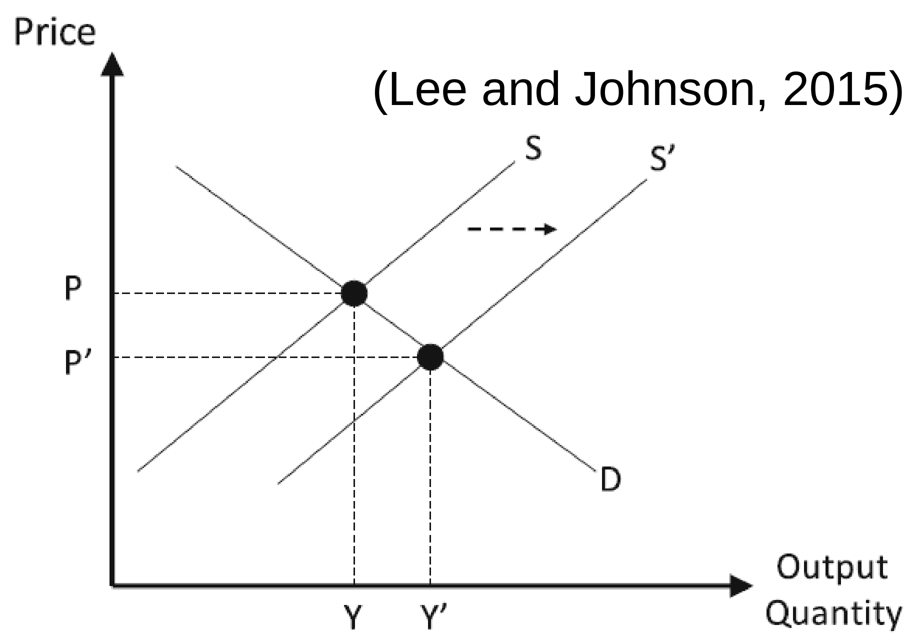



(Lee and Johnson, 2015)

# Remaining Issues in literatures…

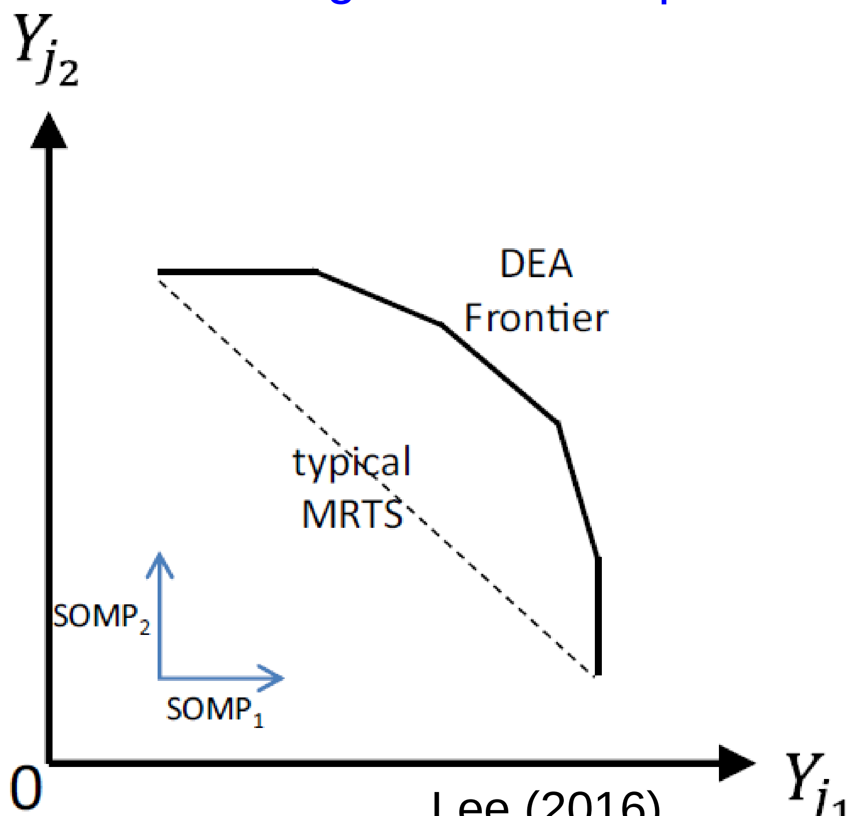
- Issue 3: in some energy market is an imperfectly competitive market (eg. oligopoly) and the market price can be affected by the total supply which is generated by all firms in the market. That is, the market price is endogenous (Hobbs and Pang, 2007; Lee and Johnson, 2015).



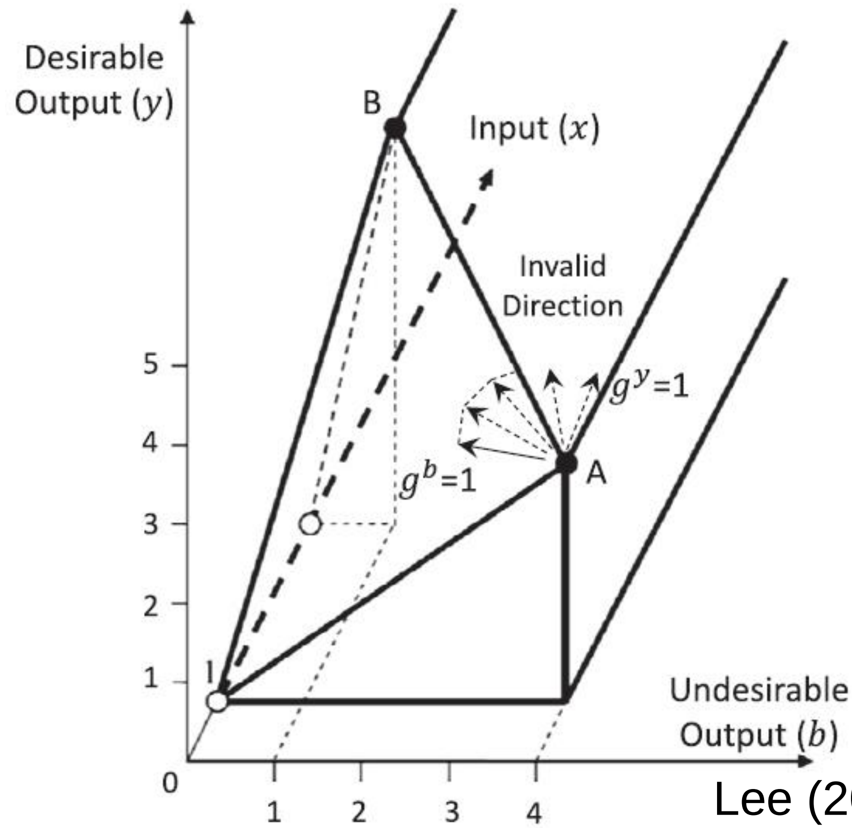




(Lee and Johnson, 2015)

- Issue 4: estimating marginal product of each undesirable output separately leads to an overestimation of marginal product and an underestimation of shadow price (i.e., MAC).
  - That is, estimating the shadow price of $SO_2$ is independent of estimating the shadow price of $NO_x$. (Lee and Zhou, 2015)



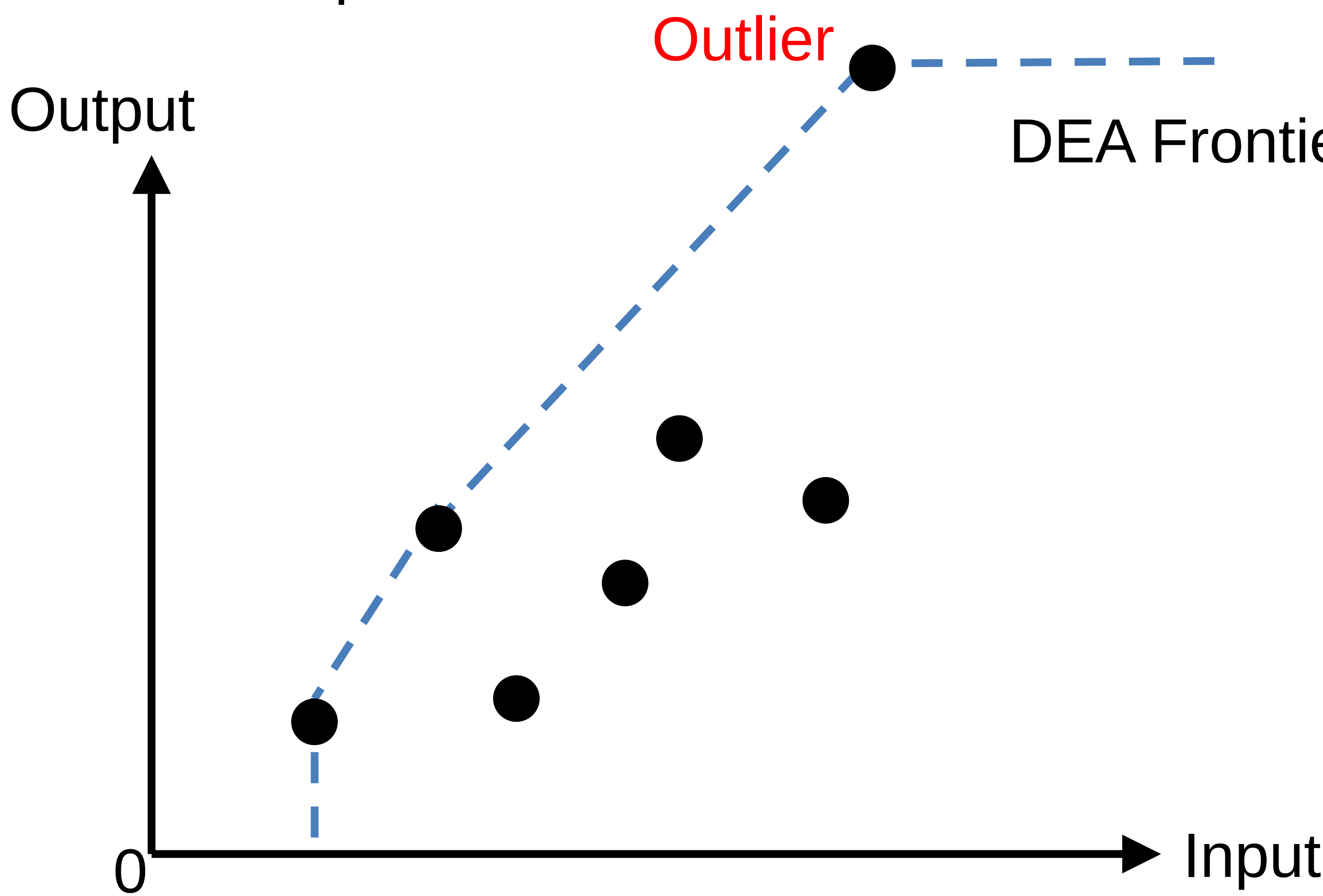

Lee (2016)




Lee (2014)

# Remaining Issues in literatures…

- Issue 5: the deterministic frontier (eg. data envelopment analysis, DEA) sensitive to the outlier and thus bias the estimate of shadow price



# Thus…

# Divide-and-Conquer

| ISSUE | | SOLUTION |
|---|---|---|
| 1 Direction-specific | ← | **Nash Equilibrium**<br>1. Nash Direction<br>2. Only One Benchmark<br>3. Noncooperative Game |
| 2 Average Calculation | ← | |
| 3 Endogenous Price in Oligopoly | ← | |
| 4 Output Estimated Separately | ← | Directional Shadow Price (DSP) |
| 5 Deterministic Frontier (DEA) | ← | Stochastic Frontier (StoNED) |

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

# Nash marginal abatement cost estimation of air pollutant emissions using the stochastic semi-nonparametric frontier

Chia-Yen Lee[a,*], Ke Wang[b,c]

[a] Institute of Manufacturing Information and Systems, National Cheng Kung University, Tainan City 701, Taiwan
[b] Center for Energy and Environmental Policy Research & School of Management and Economics, Beijing Institute of Technology, Beijing, China
[c] Sustainable Development Research Institute for Economy and Society of Beijing, Beijing, China

A R T I C L E I N F O

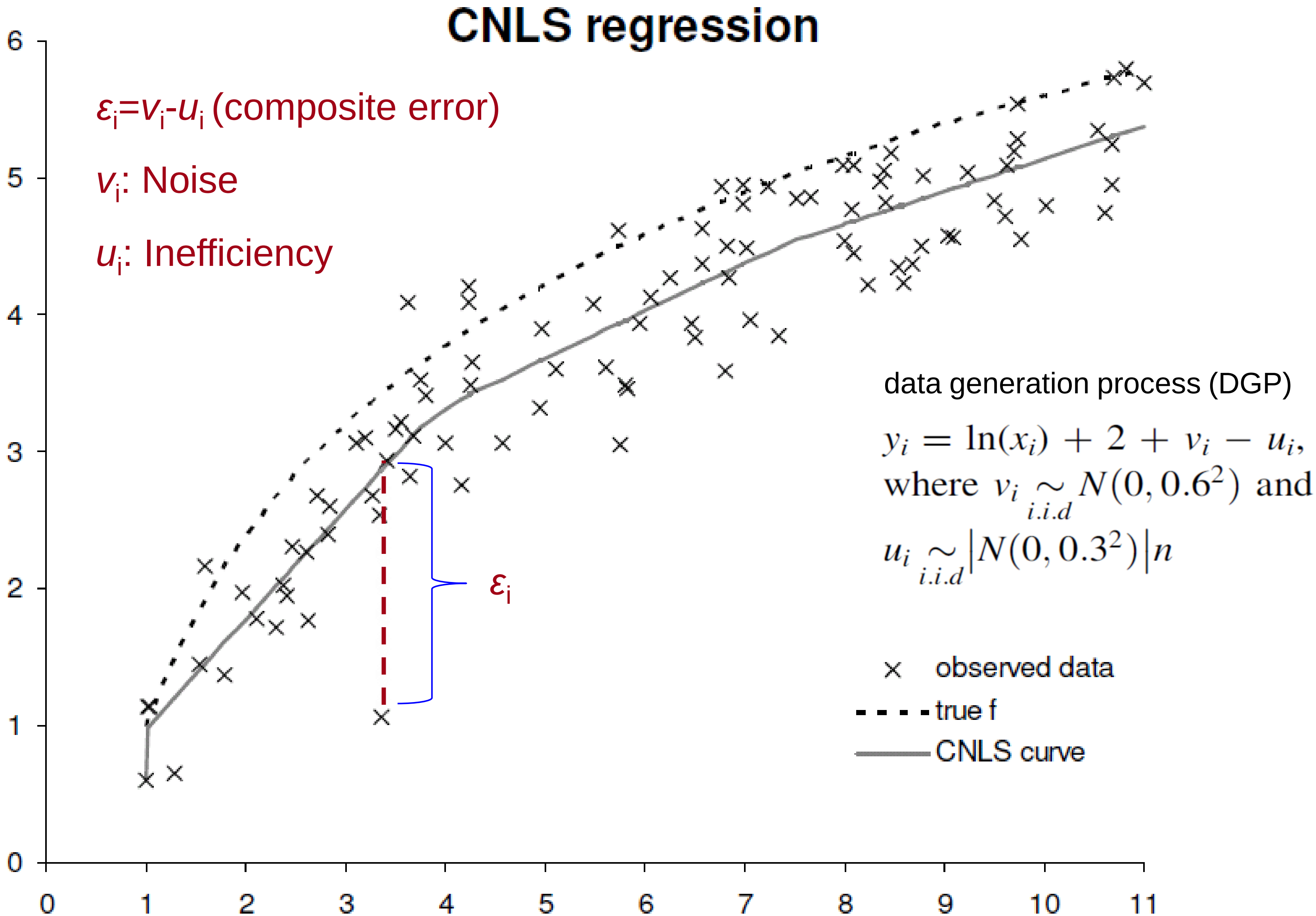
*Article history:*
Received 23 February 2018
Accepted 14 August 2018
Available online 25 August 2018

*Keywords:*
Data envelopment analysis
Marginal abatement costs
Emissions trading
Nash equilibrium
Stochastic semi-nonparametric frontier

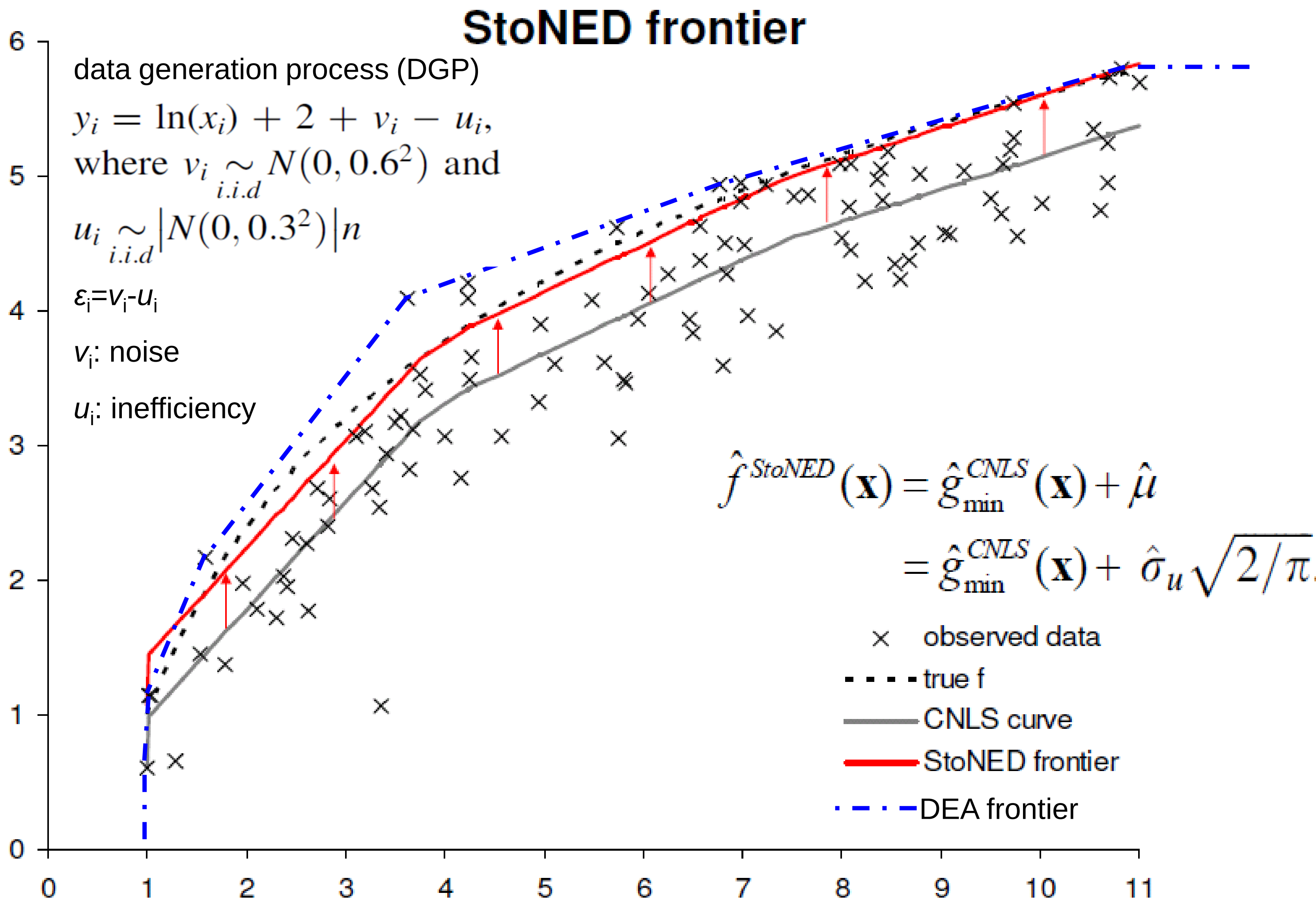
A B S T R A C T

Emissions trading (or cap and trade) is a market-based approach providing economic incentives for achieving reductions in the emissions of pollutants. Marginal abatement costs (MAC), also termed shadow prices of air pollution emissions, provide valuable guidelines to support environmental regulatory policies for $CO_2$, $SO_2$ and $NO_x$, the key contributors to climate change, smog, and acid rain. This study estimates the marginal abatement cost of undesirable outputs with respect to the Nash equilibrium on the stochastic semi-nonparametric envelopment of data (StoNED) in an imperfectly competitive market. Considering an endogenous price function of electricity, the mixed complementarity problem (MiCP) is formulated to identify the Nash equilibrium in a production possibility set. The proposed model addresses the four issues of MAC estimation in the existing literature. Applying the proposed method to an empirical study of 33 coal-fired power plants operating in China in 2013 shows that StoNED provides a robust frontier that is not sensitive to the outlier and the proposed interval of MAC estimation validates the shadow prices corresponding to the Nash equilibrium in an imperfectly competitive market.

# Graphical Illustration of CNLS




Kuosmanen and Kortelainen (2012)




(Kuosmanen and Kortelainen, 2012)

# Stochastic Frontier

- StoNED (Kuosmanen and Kortelainen, 2012)
  - Stochastic Nonparametric Envelopment of Data (StoNED) uses a composed error term to model both inefficiency and noise without assuming a functional form and assuming only monotonicity and convexity.
- Step1: Convex Nonparametric Least Square estimates $E(y_i|\mathbf{x}_i)$

$$\min_{\alpha,\beta,\varepsilon}\left\{\sum_{l=1}^{n}\varepsilon_i^2 \middle| \begin{array}{l} y_i = \alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i + \varepsilon_i \;\; \forall i = 1,\ldots,n; \\ \alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i \le \alpha_h + \boldsymbol{\beta}_h'\mathbf{x}_i \;\; \forall h,i = 1,\ldots,n; \\ \boldsymbol{\beta}_i \ge \mathbf{0} \;\; \forall i = 1,\ldots,n \end{array}\right\}$$

Objective: least square estimator
1st const: regression line
2nd const: concavity (Afriat's inequalities)
3rd const: monotonicity

- Step2: Estimation of the expected inefficiency

$$\hat{f}^{CNLS}(\mathbf{x}_i) = \hat{\alpha}_i + \hat{\boldsymbol{\beta}}_i'\mathbf{x}_i = y_i - \hat{\varepsilon}_i^{CNLS}$$

- Step3: Estimating the StoNED frontier production function
  - Shift the estimated curve upward by expected inefficiency $\mu$.

$$\hat{f}^{StoNED}(\mathbf{x}) = \hat{g}_{\min}^{CNLS}(\mathbf{x}) + \hat{\mu} = \hat{g}_{\min}^{CNLS}(\mathbf{x}) + \hat{\sigma}_u\sqrt{2/\pi}.$$

## Step2: Estimation of the expected inefficiency

- Apply the method of moments to the CNLS residual $\varepsilon_i^{CNLS} = v_i - u_i$ to estimate the expected value of inefficiency $\mu$. (Aigner et al., 1977)

- We know $\sum_{i=1}^{n} \hat{\varepsilon}_i^{CNLS} = 0$, and the second and the third central moment

$$\hat{M}_2 = \sum_{i=1}^{n} (\hat{\varepsilon}_i^{CNLS})^2 / (n-1)$$

$$\hat{M}_3 = \sum_{i=1}^{n} (\hat{\varepsilon}_i^{CNLS})^3 / (n-1)$$

- We assume $u_i \sim N^+(0, \sigma_u^2)$ and $v_i \sim N(0, \sigma_v^2)$, then they are

$$M_2 = \left[\frac{\pi - 2}{\pi}\right]\sigma_u^2 + \sigma_v^2$$

$$M_3 = \left(\sqrt{\frac{2}{\pi}}\right)\left[1 - \frac{4}{\pi}\right]\sigma_u^3$$

$$\Rightarrow \quad \hat{\sigma}_u = \sqrt[3]{\frac{\hat{M}_3}{\left(\sqrt{\frac{2}{\pi}}\right)\left[1 - \frac{4}{\pi}\right]}} \quad \Rightarrow \quad \hat{\sigma}_v = \sqrt{\hat{M}_2 - \left[\frac{\pi - 2}{\pi}\right]\hat{\sigma}_u^2}$$

- Step3: Estimating the StoNED frontier production function
  - Shift the estimated curve upward by expected inefficiency $\mu$.
  - Due to the multiple solutions of CNLS, estimate the minimum function (i.e., Minimum extrapolation)

$$\hat{g}_{\min}^{CNLS}(\mathbf{x}) = \min_{\alpha,\boldsymbol{\beta}} \left\{ \alpha + \boldsymbol{\beta}'\mathbf{x} \,\middle|\, \alpha + \boldsymbol{\beta}'\mathbf{x}_i \geq \hat{g}^{CNLS}(\mathbf{x}_i) \;\forall i = 1,\ldots,n \right\}$$

  - Adjust the minimum function by adding the expected inefficiency $\mu$ to estimate the frontier using

$$\hat{f}^{StoNED}(\mathbf{x}) = \hat{g}_{\min}^{CNLS}(\mathbf{x}) + \hat{\mu} \;= \hat{g}_{\min}^{CNLS}(\mathbf{x}) + \hat{\sigma}_u \sqrt{2/\pi}.$$

# Nash Equilibrium Identified on StoNED Frontier

## Nash Profit Function in StoNED with Undesirable Output

$$\max_{y_r, b_{qr}, x_{ir}} \left\{ \left(P^{Y_0} - \kappa \hat{Y}\right) y_r - \sum_{i \in I} P_i^X x_{ir} \;\middle|\; \begin{array}{c} y_r \le \hat{\alpha}_k + \sum_{i \in I} \hat{\beta}_{ik} x_{ir} + \sum_{q \in Q} \hat{\gamma}_{qk} b_{qr} + \hat{\mu}, \forall k \in K \\ \hat{\alpha}_k + \sum_{i \in I} \hat{\beta}_{ik} x_{ir} + \hat{\mu} \ge 0,\ \forall\, k \in K \\ \hat{B}_q \le \hat{B}_q^{CAP}, \forall q \in Q; \\ x_{ir}, y_r, b_{qr} \ge 0, \forall i \in I, q \in Q \end{array} \right\}$$

StoNED Frontier

- Objective Function: maximizing profit
- 1st constraint: concave and monotone regression line
- 2nd constraint: weak disposability (Shephard, 1974)
  - Weak disposability of desirable outputs and undesirable outputs
  - Given $(\boldsymbol{x}, y, \boldsymbol{b}) \in T$, if $0 \le \rho \le 1$, then $(\boldsymbol{x}, \rho y, \rho \boldsymbol{b}) \in T$.
- 3rd constraint: CAP
- For unique solution: $\left(P^{Y_0} - \kappa \hat{Y}\right) y_r - \sum_{i \in I} P_i^X x_{ir} \boxed{- \epsilon \sum_{q \in Q} b_{qr}}$

  where $\epsilon$ is a very small positive number

# Nash Equilibrium Identified on StoNED Frontier

POLab

## Lagrangian Function

- $L_r\left(x_{ir}, y_r, b_{qr}, \varphi 1_{kr}, \varphi 2_{kr}, \varphi 3_{qr}\right) := (P^{Y_0} - \kappa \hat{Y}) y_r - \sum_{i \in I} P_i^X x_{ir} - \epsilon \sum_{q \in Q} b_{qr} - \sum_k \varphi 1_{kr} \left(y_r - \hat{\alpha}_k - \sum_{i \in I} \hat{\beta}_{ik} x_{ir} - \sum_{q \in Q} \hat{\gamma}_{qk} b_{qr} - \hat{\mu}\right) - \sum_k \varphi 2_{kr} \left(-\hat{\alpha}_k - \sum_{i \in I} \hat{\beta}_{ik} x_{ir} - \hat{\mu}\right) - \sum_q \varphi 3_q \left(\hat{B}_q - \hat{B}_q^{CAP}\right)$

  where $\varphi 1_{kr}, \varphi 2_{kr}$, and $\varphi 3_{qr}$ are the Lagrange multipliers

## Mixed Complementarity Problem (MiCP)

- $0 \leq x_{ir} \perp -P_i^X + \sum_k \varphi 1_{kr} \hat{\beta}_{ik} + \sum_k \varphi 2_{kr} \hat{\beta}_{ik} \leq 0, \quad \forall i, r$
- $0 \leq y_r \perp P^{Y_0} - \kappa \hat{Y} - \kappa y_r - \sum_k \varphi 1_{kr} \leq 0, \quad \forall r$
- $0 \leq b_{qr} \perp -\epsilon + \sum_k \varphi 1_{kr} \hat{\gamma}_{qk} - \varphi 3_q \leq 0, \quad \forall q, r$
- $0 \leq \varphi 1_{kr} \perp y_r - \hat{\alpha}_k - \sum_{i \in I} \hat{\beta}_{ik} x_{ir} - \sum_{q \in Q} \hat{\gamma}_{qk} b_{qr} - \hat{\mu} \leq 0, \quad \forall k, r$
- $0 \leq \varphi 2_{kr} \perp -\hat{\alpha}_k - \sum_{i \in I} \hat{\beta}_{ik} x_{ir} - \hat{\mu} \leq 0, \quad \forall k, r$
- $0 \leq \varphi 3_q \perp \hat{B}_q - \hat{B}_q^{CAP} \leq 0, \quad \forall q$

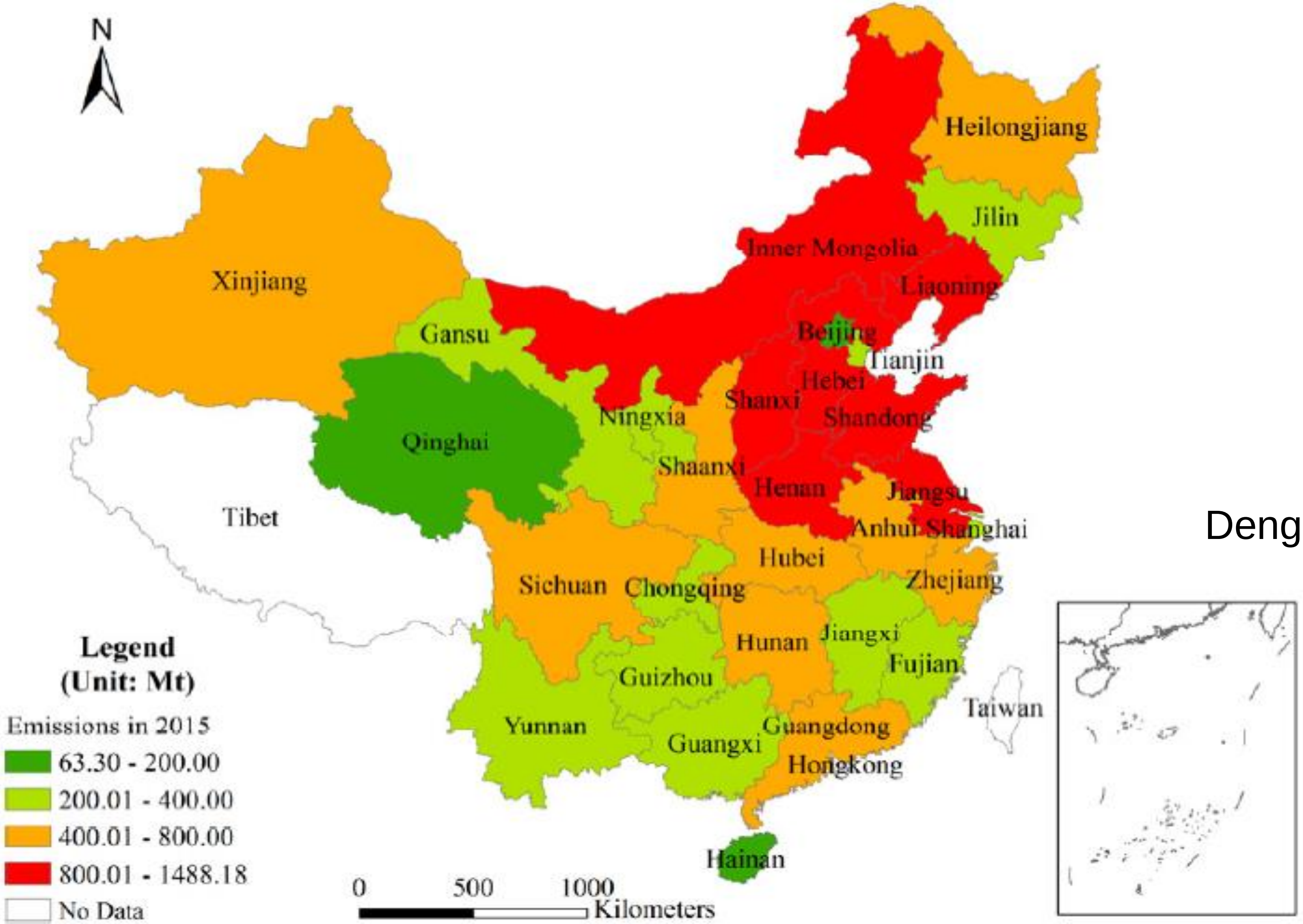
## This Study…

Identifies the Nash equilibrium on stochastic frontier and estimate the MAC with respect to this efficient benchmark

to address 5 issues mentioned above...

# Empirical Study

- Emission intensity distribution of CO2 in 2015;

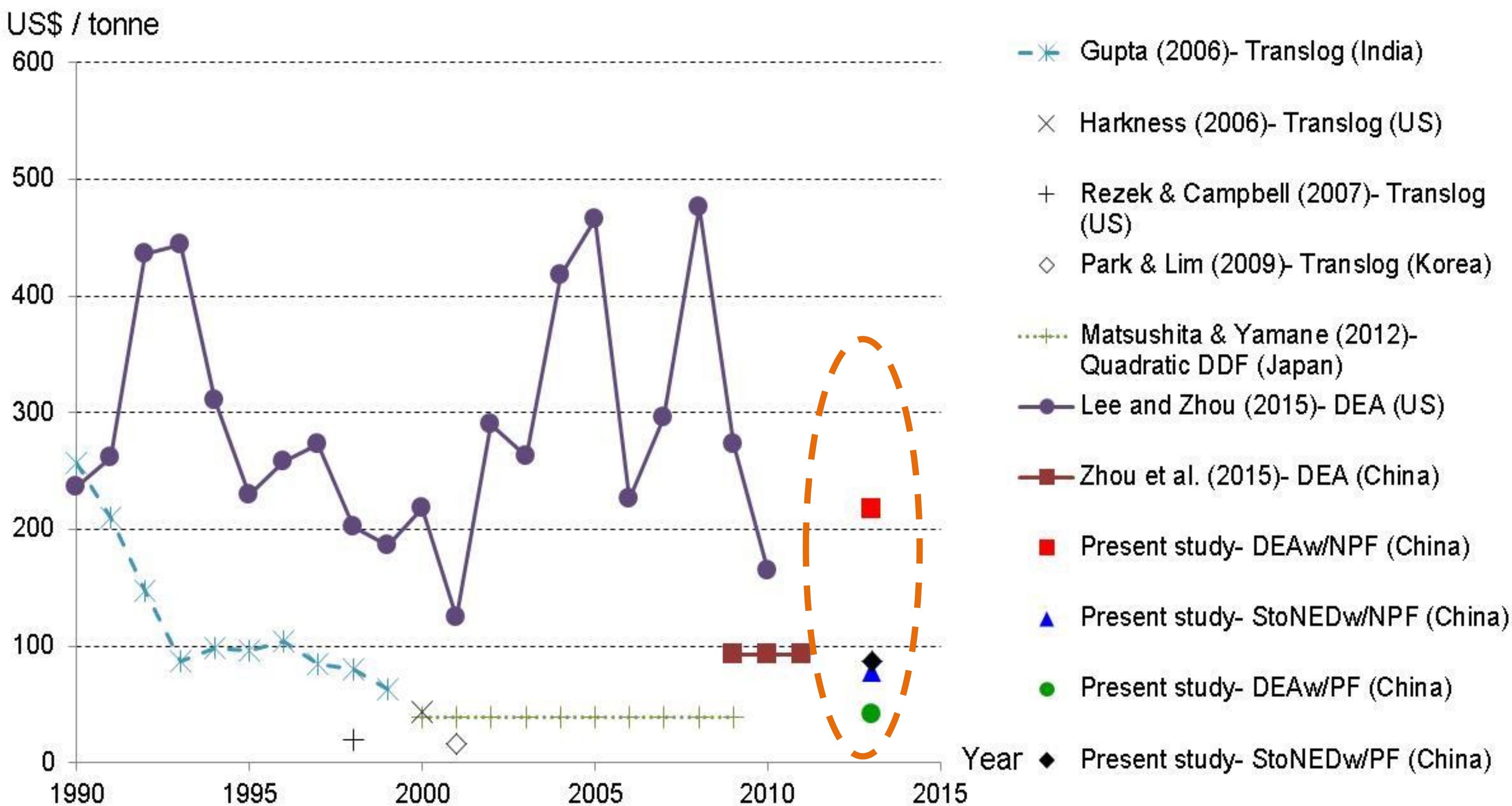



Deng et al. (2015)

- 2013 China Coal Power Plants in North and Northeast regions
  - North region includes province Beijing, Tianjin, Hebei, Shanxi, Shandong, and Inner Mongolia while Northeast region includes Liaoning, Jilin, and Heilongjiang.
  - 33 coal-fired power plants with nameplate capacity larger than 1MkW
- Data Source: China Electric Power Yearbook 2014
- Inputs and Outputs
  - One desirable output: the annual amount of electricity generated by coal in Megawatt-hours (MWh)
  - Three undesirable outputs: the annual amount in tons of $CO_2$, $SO_2$ and $NO_x$
  - One input: the annual amount in tons of coal consumption
  - The inverse demand function: $P^Y(\hat{Y}) := 4.5 \times 10^7 - 123\hat{Y}$

POLab

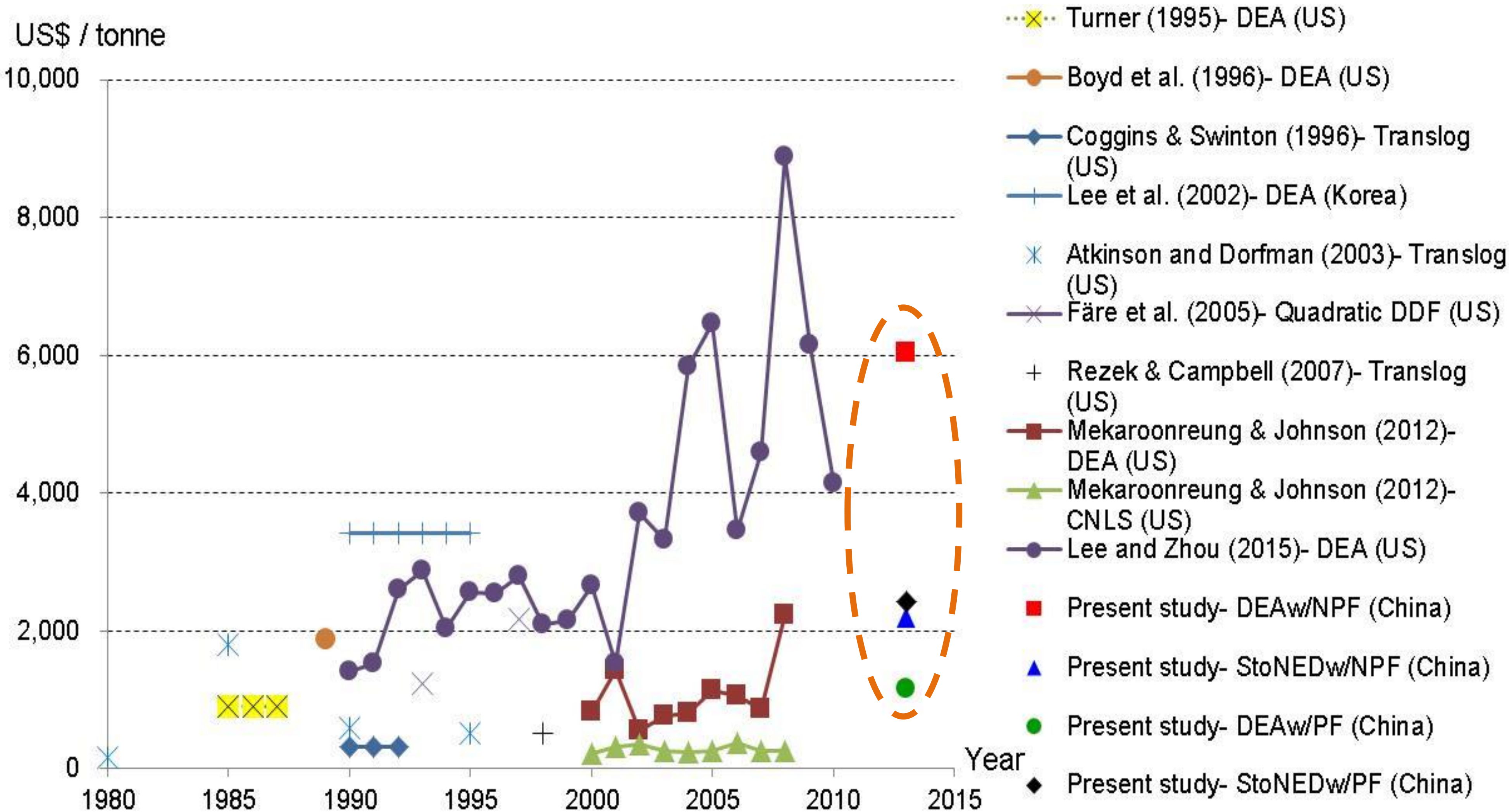
- Comparison of the MAC of $CO_2$



# Empirical Study

- Comparison of the MAC of $SO_2$

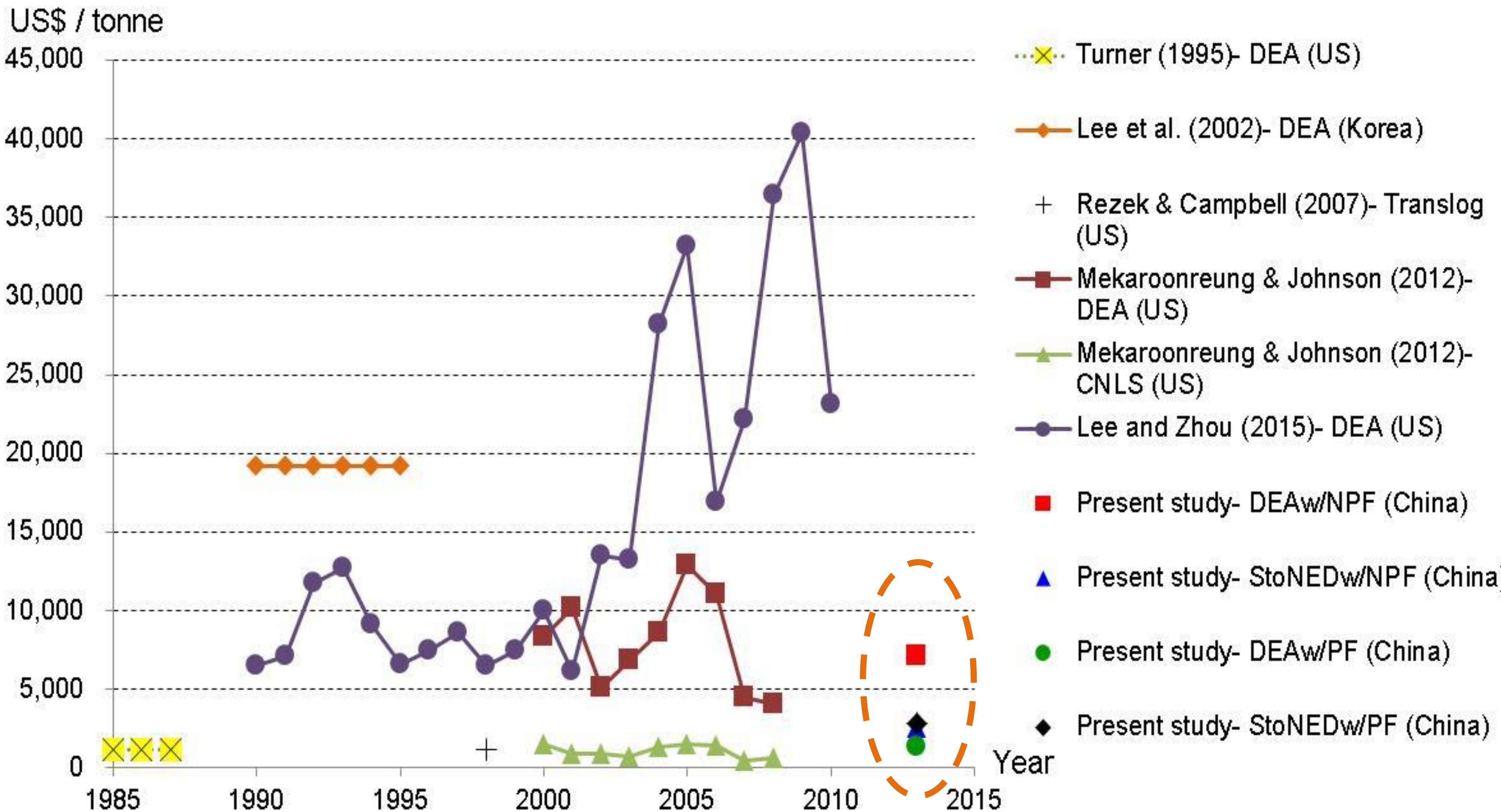
**Relatively higher than literatures…**



# Empirical Study

- Comparison of the MAC of $NO_x$

**Relatively lower than literatures…**



# Nash Marginal Abatement Costs

POLab

European Journal of Operational Research 273 (2019) 390–400

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

ELSEVIER

Decision Support

## Nash marginal abatement cost estimation of air pollutant emissions using the stochastic semi-nonparametric frontier

Check for updates

Chia-Yen Lee [a,*], Ke Wang [b,c]

[a] *Institute of Manufacturing Information and Systems, National Cheng Kung University, Tainan City 701, Taiwan*
[b] *Center for Energy and Environmental Policy Research & School of Management and Economics, Beijing Institute of Technology, Beijing, China*
[c] *Sustainable Development Research Institute for Economy and Society of Beijing, Beijing, China*

ARTICLE INFO

*Article history:*
Received 23 February 2018
Accepted 14 August 2018
Available online 25 August 2018

*Keywords:*
Data envelopment analysis
Marginal abatement costs
Emissions trading
Nash equilibrium
Stochastic semi-nonparametric frontier

ABSTRACT

Emissions trading (or cap and trade) is a market-based approach providing economic incentives for achieving reductions in the emissions of pollutants. Marginal abatement costs (MAC), also termed shadow prices of air pollution emissions, provide valuable guidelines to support environmental regulatory policies for $CO_2$, $SO_2$ and $NO_x$, the key contributors to climate change, smog, and acid rain. This study estimates the marginal abatement cost of undesirable outputs with respect to the Nash equilibrium on the stochastic semi-nonparametric envelopment of data (StoNED) in an imperfectly competitive market. Considering an endogenous price function of electricity, the mixed complementarity problem (MiCP) is formulated to identify the Nash equilibrium in a production possibility set. The proposed model addresses the four issues of MAC estimation in the existing literature. Applying the proposed method to an empirical study of 33 coal-fired power plants operating in China in 2013 shows that StoNED provides a robust frontier that is not sensitive to the outlier and the proposed interval of MAC estimation validates the shadow prices corresponding to the Nash equilibrium in an imperfectly competitive market.

# Nash Allocation of Emission Permits

Lee, Chia-Yen. 2019. Decentralized Allocation of Emission Permits by Nash Data Envelopment Analysis in the Coal-Fired Power Market. Journal of Environmental Management, 241, 353-362.

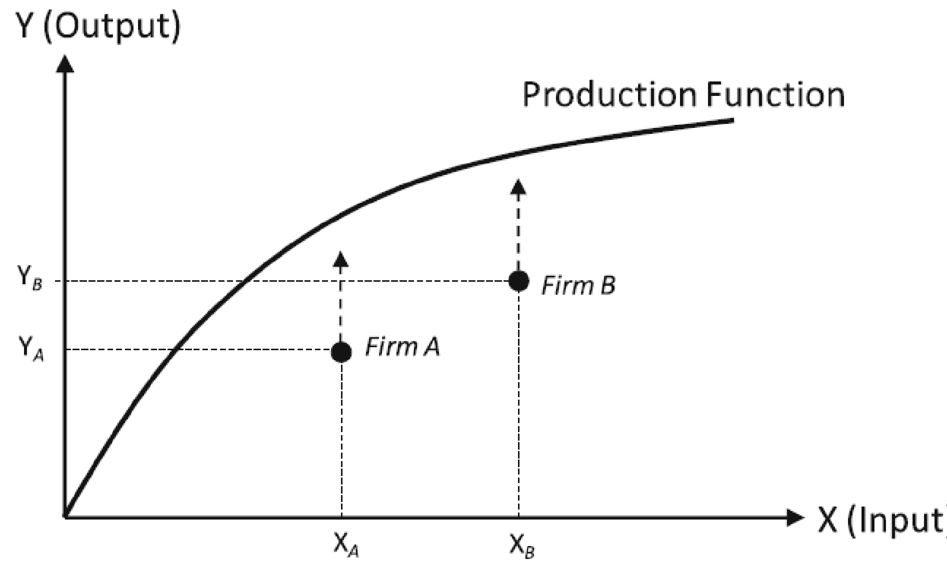
# Outline

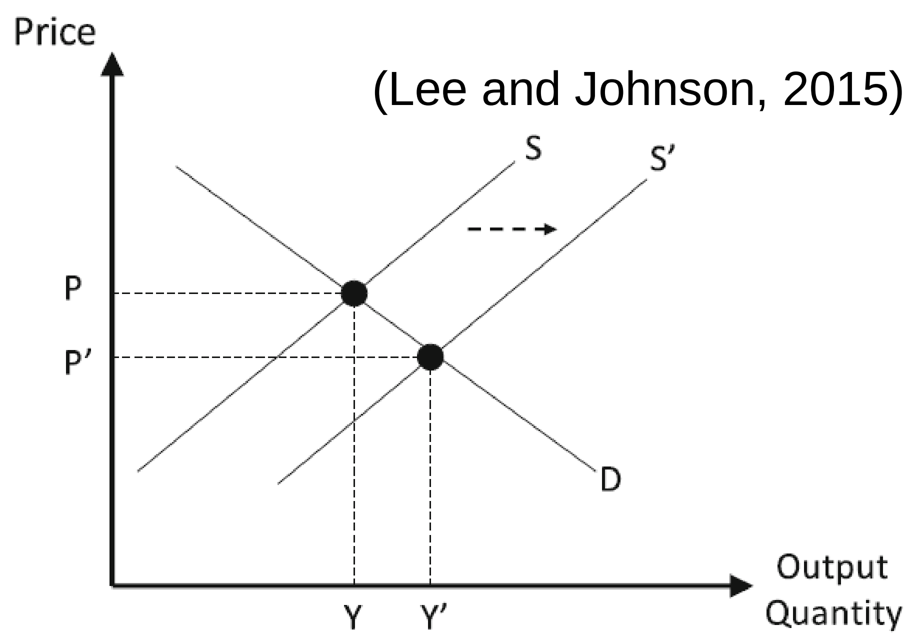
- Introduction
- Allocation of Emission Permit
- Centralized AEP Model in Perfectly Competitive Market
  - Lozano Model
  - Feng model
- Decentralized AEP Model in Imperfectly Competitive Market
  - Nash CRS Model
  - Nash NIRS model
- Empirical Study- Coal-fired Power Plant in China
- Conclusions

# Introduction

## Typical Assumption in DEA

- All firms want to be as productive as possible
- An exogenous price in a perfectly competitive market (Cherchye, Kuosmanen, & Post, 2002)
- DMU are independent with each other



# Introduction

- **Oligopolistic markets**
  - An inefficient firm that increases output may reduce overall profits by increasing the market quantity and causing the market price to fall (Johnson and Ruggiero, 2011).

- **Rational inefficiency**
  - A firm is maximizing its profit by intentionally operating at lower productivity levels (Lee and Johnson, 2015)
    - non-cooperative game (Nash, 1951) (Lee & Johnson, 2015)
    - a firm may overestimate the revenue when expanding output by assuming exogenous price (Lee, 2016).

  - Energy markets are imperfectly competitive
    - market price can be affected by the total supply which is generated by all firms in the market. That is, the market price is endogenous (Hobbs and Pang, 2007; Lee and Johnson, 2015).

# Introduction

- This study considers the allocation of emission permits (AEP) at the coal-fired power plants operating in China in 2013
  - North and the Northeast regions

- Air Pollution in China
  - In 2012 China was the largest contributor to carbon emissions from fossil fuel burning and cement production, and responsible for 25 percent of global carbon emissions.
  - manufacturing and power generation are the major sectors contributing to China's carbon emissions, together these sectors accounted for 85 percent of China's total carbon emissions in 2012 (Liu, 2015).
  - Since 2013, seven pilot provinces and provincial cities, i.e. Shenzhen, Shanghai, Beijing, Guangdong, Tianjin, Chongqing and Hubei, have successively launched their emission trading scheme.

# Literatures of AEP by DEA

- Gomes and Lins (2008) developed zero sum gains DEA (ZSG-DEA) models and reallocate the CO2 emissions among the 64 signatory countries of the Kyoto protocol. Each DMU adjusts CO2 emissions regarded as input to ensure all the DMUs become efficient after reallocation.
- Lozano et al. (2009) implemented the centralized AEP model by using three objectives separately: maximizing aggregated desirable production, minimizing undesirable total emissions, and minimizing the consumption of input resources. The approach was applied to 41 plants from the Swedish pulp and paper industry.
- Feng et al. (2015) proposed maximizing the total potential gross domestic product (GDP) in the first stage and then provided two compensate schemes according to their AEP contributions to allocate a whole estate to different claimants fairly.
- Ji et al. (2016) proposed a multicriteria centralized model for AEP in large data sets. They considered the emission standard as a control variable, and finds its optimal value together with each DMU's optimal emission permits. The model is applied to allocate $SO_2$ emission permits in the 202 Chinese prefecture-level cities.

# Challenges in Literatures

- However…
  - the perfect cooperation assumption may not be applicable to the real practice, and thus the result presented a fair but too ideal allocation of the CO2 emission.
  - They also treated the undesirable outputs as inputs and didn't consider the weak disposability between good outputs and bad outputs.
  - Their approach benefits the technical and environmental efficiency; however, it cannot be justified from the allocative efficiency or economic efficiency without price information.

- This study…
  - How about decentralized AEP model?
  - How to optimize the AEP by DEA in energy market with endogenous price?
  - How about efficiency estimation before and after AEP

- Lozano Model- Three-phase CRS model (Färe's weak disposability)
  - maximize the aggregated electricity generation
  - minimize the total emission of undesirable output
  - minimize the use of variable inputs of the plants

First phase of Lozano model:

Max $\gamma$

s.t $\sum_{k\in K}\lambda_{kr}X_{ik} \le x_{ir}, \forall i \in I_v, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}X_{ik} \le X_{ir}, \forall i \in I_f, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}Y_k \ge y_r, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}B_k = B_r - \Delta b_r, \forall r \in K;$

$x_{ir} \le X_{ir}, \forall i \in I, \forall r \in K;$

$y_r \ge Y_r, \forall r \in K;$

$\sum_{r\in K} y_r \ge \gamma \sum_{r\in K} Y_r;$ **(output generation larger than current level)**

$\sum_{r\in K}\Delta b_r = C;$ **(emission reduction target)**

$x_{ir}, y_r, B_r - \Delta b_r, \lambda_{kr} \ge 0, \forall i \in I, \forall k \in K, \forall r \in K.$

- Lozano Model- Three-phase CRS model (Färe's weak disposability)
  - maximize the aggregated electricity generation
  - minimize the total emission of undesirable output
  - minimize the use of variable inputs of the plants

Second phase of Lozano model:

Min $\kappa$

s.t $\sum_{k\in K}\lambda_{kr}X_{ik} \le x_{ir}, \forall i \in I_v, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}X_{ik} \le X_{ir}, \forall i \in I_f, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}Y_k \ge y_r, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr}B_k = B_r - \Delta b_r, \forall r \in K;$

$x_{ir} \le X_{ir}, \forall i \in I, \forall r \in K;$

$y_r \ge Y_r, \forall r \in K;$

$\sum_{r\in K} y_r \ge \sum_{r\in K} y_r^*$ **(given output generation from 1st phase)**

$\sum_{r\in K}(B_r - \Delta b_r) \le \kappa \sum_{r\in K} B_r;$ **(emission reduction lower than current level)**

$\sum_{r\in K}\Delta b_r = C;$

$x_{ir}, y_r, B_r - \Delta b_r, \lambda_{kr} \ge 0, \forall i \in I, \forall k \in K, \forall r \in K;$

$0 \le \kappa \le 1.$

# Centralized AEP Model in Perfectly Competition

POLab

- Lozano Model- Three-phase CRS model (Färe's weak disposability)
  - maximize the aggregated electricity generation
  - minimize the total emission of undesirable output
  - minimize the use of variable inputs of the plants

Third phase of Lozano model:

$$\text{Min } \frac{1}{|I_v|}\sum_{i\in I_v}\sum_{r\in K}\omega_{ir}$$

$$\text{s.t } \sum_{k\in K}\lambda_{kr}X_{ik} \le x_{ir}, \forall i \in I_v, \forall r \in K;$$

$$\sum_{k\in K}\lambda_{kr}X_{ik} \le X_{ir}, \forall i \in I_f, \forall r \in K;$$

$$\sum_{k\in K}\lambda_{kr}Y_k \ge y_r, \forall r \in K;$$

$$\sum_{k\in K}\lambda_{kr}B_k = B_r - \Delta b_r, \forall r \in K;$$

$$x_{ir} = \omega_{ir}X_{ir}, \forall i \in I_v, \forall r \in K; \ y_r \ge Y_r, \forall r \in K;$$ **(reduce using the variable inputs)**

$$\sum_{r\in K}y_r \ge \sum_{r\in K}y_r^*$$ **(given output generation from 1st phase)**

$$\sum_{r\in K}(B_r - \Delta b_r) \le \kappa^*\sum_{r\in K}B_r;$$ **(given emission reduction from 2nd phase)**

$$\sum_{r\in K}\Delta b_r = C;$$

$$x_{ir}, y_r, B_r - \Delta b_r, \lambda_{kr} \ge 0, \forall i \in I, \forall k \in K, \forall r \in K;$$

$$0 \le \omega_{ir} \le 1, \forall i \in I_v, \forall r \in K;$$

# Centralized AEP Model in Perfectly Competition

## Feng Model- Two-phase VRS model (Färe's weak disposability)

- treats all inputs as fixed inputs to suggest a conservative solution; that is, input factors are not adjustable in Feng model.
- 1st phase: find the parameters of upper bound $\Delta B_r^+$ and lower bound $\Delta B_r^-$ of $\Delta b_r$ are pre-determined
- 2nd phase: Maximize the desirable outputs

The lower bound $\Delta B_r^-$ of Feng model:

Max $\sum_{k\in K} \lambda_{kr} B_k$

s.t $\sum_{k\in K} \lambda_{kr} X_{ik} \leq \xi_r X_{ir}, \forall i \in I;$

$\sum_{k\in K} \lambda_{kr} Y_k \geq \theta_r Y_r;$

$\sum_{k\in K} \lambda_{kr} = \xi_r;$

$\epsilon \leq \xi_r \leq 1;$

$\theta_r > 0;$

$\lambda_{kr} \geq 0, \forall k \in K$

Zhou et al. (2008)

- based on the nonnegativity property, i.e., $B_r - \Delta b_r \geq 0$, the upper bound $\Delta B_r^+$ is equal to $B_r$. The lower bound is $\Delta B_r^- = B_r - \sum_{k\in K} \lambda_{kr}^* B_k$.

# Centralized AEP Model in Perfectly Competition

## Feng Model- Two-phase VRS model (Färe's weak disposability)

- treats all inputs as fixed inputs to suggest a conservative solution; that is, input factors are not adjustable in Feng model.
- 1st phase: find the parameters of upper bound $\Delta B_r^+$ and lower bound $\Delta B_r^-$ of $\Delta b_r$ are pre-determined
- 2nd phase: Maximize the desirable outputs

The AEP of Feng model:

Max $\sum_{r\in K}\sum_{k\in K}\lambda_{kr}Y_k$

s.t. $\sum_{k\in K}\lambda_{kr}X_{ik} \le \xi_r X_{ir}, \forall i \in I, r \in K;$

$\sum_{k\in K}\lambda_{kr}B_k = B_r - \Delta b_r, \forall r \in K;$

$\sum_{k\in K}\lambda_{kr} = \xi_r, \forall r \in K;$

$\sum_{r\in K}\Delta b_r = C;$ **(emission reduction target)**

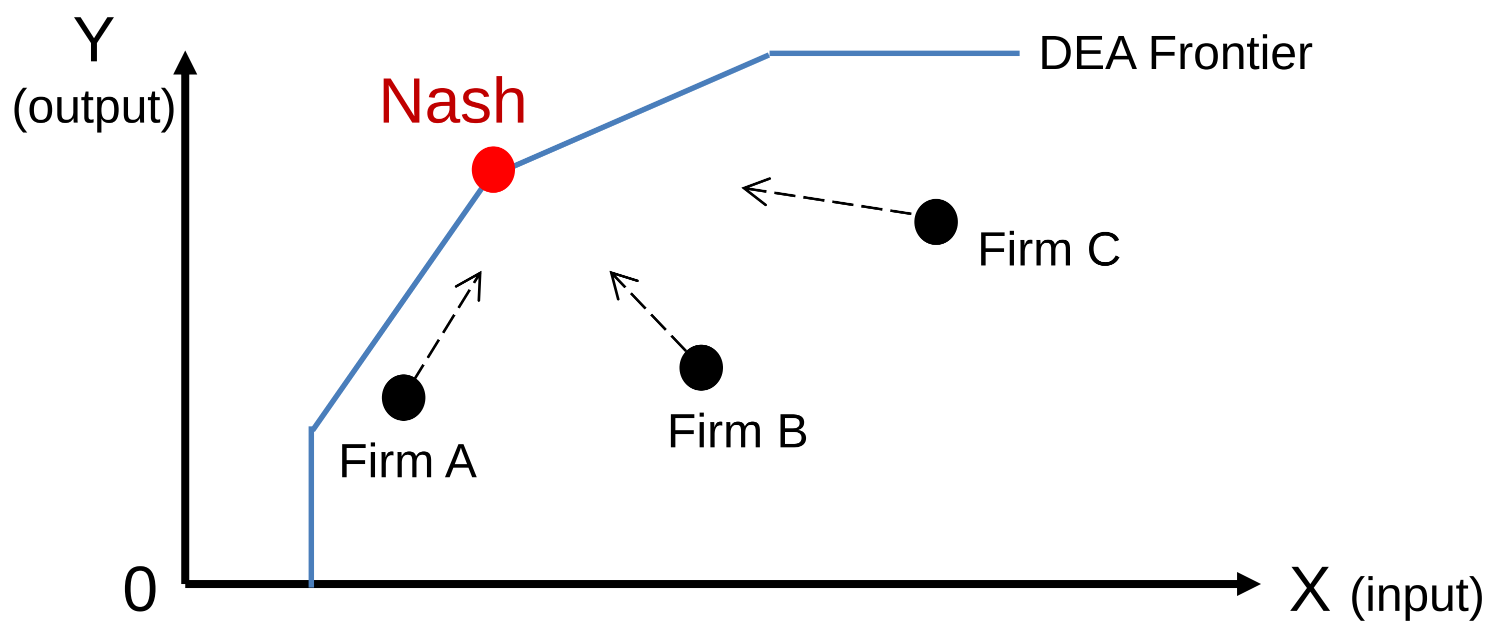
$\Delta b_r$ unrestricted and $\Delta b_r \in [\Delta B_r^-, \Delta B_r^+], \forall r \in K;$

$\epsilon \le \xi_r \le 1, \forall r \in K;$

$\lambda_{kr} \ge 0, \forall k \in K, r \in K$

## Nash Equilibrium (Lee and Johnson, 2015)

- Nash equilibrium is a solution of a **non-cooperative** game involving two or more players, and **no player has incentive** to change the strategy due to a reduction in the immediate payoff.



POLab

## Definition of Nash Equilibrium

**Definition**: Let $K$ be a finite number of players, $\theta$ a utility (or profit) function, $T_k$ a strategy set (production possibility set) for player $k = 1, \dots, |K|$, and $(\boldsymbol{x}_k, \boldsymbol{y}_k, \boldsymbol{b}_k) = \left(x_{k1}, \dots, x_{k|I|}, y_{k1}, \dots, y_{k|J|}, b_{k1}, \dots, b_{k|Q|}\right) \in T_k$ an observed production vector. A vector $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) = \left((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*, \boldsymbol{b}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*, \boldsymbol{b}_2^*), \dots, (\boldsymbol{x}_{|K|}^*, \boldsymbol{y}_{|K|}^*, \boldsymbol{b}_{|K|}^*)\right) \in T_1 \times T_2 \times \cdots \times T_{|K|}$ is called a Nash equilibrium if and only if

$$\theta(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) \geq \theta\left(\boldsymbol{x}_k, \widehat{\boldsymbol{x}}_{(-k)}^*, \boldsymbol{y}_k, \widehat{\boldsymbol{y}}_{(-k)}^*, \boldsymbol{b}_k, \widehat{\boldsymbol{b}}_{(-k)}^*\right), \forall (\boldsymbol{x}_k, \boldsymbol{y}_k, \boldsymbol{b}_k) \in T_k,$$

where $\widehat{\boldsymbol{x}}_{(-k)}^* = (\boldsymbol{x}_1^*, \dots, \boldsymbol{x}_{k-1}^*, \boldsymbol{x}_{k+1}^*, \dots, \boldsymbol{x}_{|K|}^*)$, $\widehat{\boldsymbol{y}}_{(-k)}^* = (\boldsymbol{y}_1^*, \dots, \boldsymbol{y}_{k-1}^*, \boldsymbol{y}_{k+1}^*, \dots, \boldsymbol{y}_{|K|}^*)$ and $\widehat{\boldsymbol{b}}_{(-k)}^* = (\boldsymbol{b}_1^*, \dots, \boldsymbol{b}_{k-1}^*, \boldsymbol{b}_{k+1}^*, \dots, \boldsymbol{b}_{|K|}^*)$ holds for all $k = 1, \dots, |K|$.

## Price function of the desirable output

- Inverse demand function as $P^{Y}(\hat{Y}) := P^{Y_0} - \tau\hat{Y}$
  - where $P^{Y}(\cdot) \geq 0$, $\hat{Y} = (\sum_k y_k + \bar{Y})$, $\bar{Y}$ is a constant representing the least and fixed output levels generated by the plants without market power, $P^{Y_0}$ is a positive intercept, and $\tau \geq 0$ indicates the price sensitive coefficient of the desirable output.

- The revenue function $P^{Y}(\hat{Y})y_r$ is concave

- The abatement target of the undesirable quantity $\sum_{r\in K} \Delta b_r = C$.

- Let $D$ be the minimal amount of electricity consumption for demand fulfillment by these plants with market power, i.e. $\sum_{r\in K} y_r \geq D$.

- Assume a competitive input market and the price of variable input is a constant, $P_i^X$, $\forall i \in I_v$. (i.e., coal consumption)

## Nash CRS Model (Färe's weak disposability)

- First phase: find the parameters of upper bound $\Delta B_r^+$ and lower bound $\Delta B_r^-$ of $\Delta b_r$ are pre-determined
- Second phase: Maximize the desirable outputs

The lower bound $\Delta B_r^-$ of CRS model:

$$
\begin{aligned}
&\text{Max } \textstyle\sum_{k \in K} \lambda_{kr} B_k \\
&\text{s.t } \textstyle\sum_{k \in K} \lambda_{kr} X_{ik} \leq X_{ir}, \forall i \in I_f; \\
&\textstyle\sum_{k \in K} \lambda_{kr} X_{ik} \leq x_{ir}, \forall i \in I_v; \\
&\textstyle\sum_{k \in K} \lambda_{kr} Y_k \geq y_r; \\
&\lambda_{kr}, x_{ir}, y_r \geq 0, \forall k \in K, i \in I_v
\end{aligned}
$$

- based on the nonnegativity property, i.e., $B_r - \Delta b_r \geq 0$, the upper bound $\Delta B_r^+$ is equal to $B_r$. The lower bound is $\Delta B_r^- = B_r - \sum_{k \in K} \lambda_{kr}^* B_k$.

## Nash CRS Model (Färe's weak disposability)

- Each firm should maximize its Nash profit function
- The term $\epsilon(B_r - \Delta b_r)$ is used for reducing multiple solution issue

$$\text{Max } \left(P^{Y_0} - \tau\hat{Y}\right)y_r - \sum_{i\in I_v} P_i^X x_{ir} - \epsilon(B_r - \Delta b_r)$$

$$\text{s.t } \sum_{k\in K} \lambda_{kr} X_{ik} \le X_{ir}, \forall i \in I_f;$$

$$\sum_{k\in K} \lambda_{kr} X_{ik} \le x_{ir}, \forall i \in I_v;$$

$$\sum_{k\in K} \lambda_{kr} Y_k \ge y_r;$$

$$\sum_{k\in K} \lambda_{kr} B_k = \boxed{B_r - \Delta b_r;}$$

$$\sum_{r\in K} y_r \ge D;$$

$$\boxed{\sum_{r\in K} \Delta b_r = C;}$$

$$\Delta b_r \text{ unrestricted and } \Delta b_r \in [\Delta B_r^-, \Delta B_r^+];$$

$$x_{ir}, y_r, \lambda_{kr} \ge 0, \forall k \in K,\ i \in I_v$$

## Karush-Kuhn-Tucker (KKT) Conditions

- First order necessary conditions

*If $x^*$ is a regular point and a local minimizer of the problem*

$$\begin{array}{ll} \text{minimize} & f(x) \\ \textit{subject to} & h(x) = 0, \\ & g(x) \leq 0, \end{array}$$

*where all functions are continuously differentiable, then there exist $\lambda \in \Re^m$ and a nonnegative $\mu \in \Re^p$ such that*

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x^*) + \sum_{j=1}^{p} \mu_j \nabla g_j(x^*) = 0$$

*and*

$$\mu_j g_j(x^*) = 0,\ j = 1, \ldots, p \ \ (\textit{Complementary slackness}).$$

## Mixed Complementarity Problem (MiCP)

- $0 \le x_{ir} \perp -P_i^X + \varphi 2_{ir} \le 0, \quad \forall i \in I_v, r \in K$
- $0 \le y_r \perp P^{Y_0} - \tau \hat{Y} - \tau y_r - \varphi 3_r + \varphi 5 \le 0, \quad \forall r \in K$
- $\epsilon - \varphi 4_r + \varphi 6 + \varphi 7_r - \varphi 8_r = 0$ ($\Delta b_r$ unrestricted), $\quad \forall r \in K$
- $0 \le \lambda_{kr} \perp -\sum_{i \in I_f} \varphi 1_{ir} X_{ik} - \sum_{i \in I_v} \varphi 2_{ir} X_{ik} + \varphi 3_r Y_k - \varphi 4_r B_k \le 0, \quad \forall k, r \in K$
- $0 \le \varphi 1_{ir} \perp \sum_{k \in K} \lambda_{kr} X_{ik} - X_{ir} \le 0, \quad \forall i \in I_f, r \in K$
- $0 \le \varphi 2_{ir} \perp \sum_{k \in K} \lambda_{kr} X_{ik} - x_{ir} \le 0, \quad \forall i \in I_v, r \in K$
- $0 \le \varphi 3_r \perp y_r - \sum_{k \in K} \lambda_{kr} Y_k \le 0, \quad \forall r \in K$
- $\sum_{k \in K} \lambda_{kr} B_k - B_r + \Delta b_r = 0$ ($\varphi 4_r$ unrestricted), $\quad \forall r \in K$
- $0 \le \varphi 5 \perp D - \sum_{r \in K} y_r \le 0,$
- $C - \sum_{r \in K} \Delta b_r = 0$ ($\varphi 6$ unrestricted),
- $0 \le \varphi 7_r \perp \Delta B_r^- - \Delta b_r \le 0, \quad \forall r \in K$
- $0 \le \varphi 8_r \perp \Delta b_r - \Delta B_r^+ \le 0, \quad \forall r \in K$

# Decentralized AEP in an Imperfectly Competition

**Lemma**: Consider an imperfectly competitive market with $|K|$ firms, an inverse demand function $P^Y(\cdot)$ that is nonincreasing and continuously differentiable in $y_k$, and an inverse supply function $P^X(\cdot)$ that is nondecreasing (or in our case, a constant) and continuously differentiable in $x_k$. If the profit function $\pi_k(x_k, y_k, b_k)$ is concave and the variables $x_k, y_k, b_k \geq 0$, then $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) = \left((\boldsymbol{x}_1^*, \boldsymbol{y}_1^*, \boldsymbol{b}_1^*), (\boldsymbol{x}_2^*, \boldsymbol{y}_2^*, \boldsymbol{b}_2^*), \ldots, (\boldsymbol{x}_{|K|}^*, \boldsymbol{y}_{|K|}^*, \boldsymbol{b}_{|K|}^*)\right)$ is a Nash equilibrium solution if and only if

$\nabla_{\boldsymbol{x}_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) \leq 0$ and $\nabla_{\boldsymbol{y}_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*) \leq 0, \forall k;$

$\boldsymbol{x}_k^*\left[\nabla_{\boldsymbol{x}_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*)\right] = 0$ and $\boldsymbol{y}_k^*\left[\nabla_{\boldsymbol{y}_k}\pi_k(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{b}^*)\right] = 0, \forall k,$

where $(\boldsymbol{x}_k^*, \boldsymbol{y}_k^*, \boldsymbol{b}_k^*) \in \tilde{T}$ and $\tilde{T}$ is the PPS estimated by DEA.

**Theorem**: The proposed MiCP generates Nash equilibrium solution $(x_{ir}, y_r, B_r - \Delta b_r) \in \tilde{T}$, where $\tilde{T}$ is Färe's convex CRS technology, and satisfies the demand fulfillment constraint $\sum_{r\in K} y_r \geq D$ and the cap constraint $\sum_{r\in K} \Delta b_r = C$.

# Decentralized AEP in an Imperfectly Competition

## Nash NIRS Model (Färe's weak disposability)

- Non-increasing returns-to-scale (NIRS)

The lower bound $\Delta B_r^-$ of NIRS model:

Max $\sum_{k\in K} \lambda_{kr} B_k$

s.t $\sum_{k\in K} \lambda_{kr} X_{ik} \leq X_{ir}, \forall i \in I_f;$

$\sum_{k\in K} \lambda_{kr} X_{ik} \leq x_{ir}, \forall i \in I_v;$

$\sum_{k\in K} \lambda_{kr} Y_k \geq y_r;$

$\sum_{k\in K} \lambda_{kr} \leq 1;$

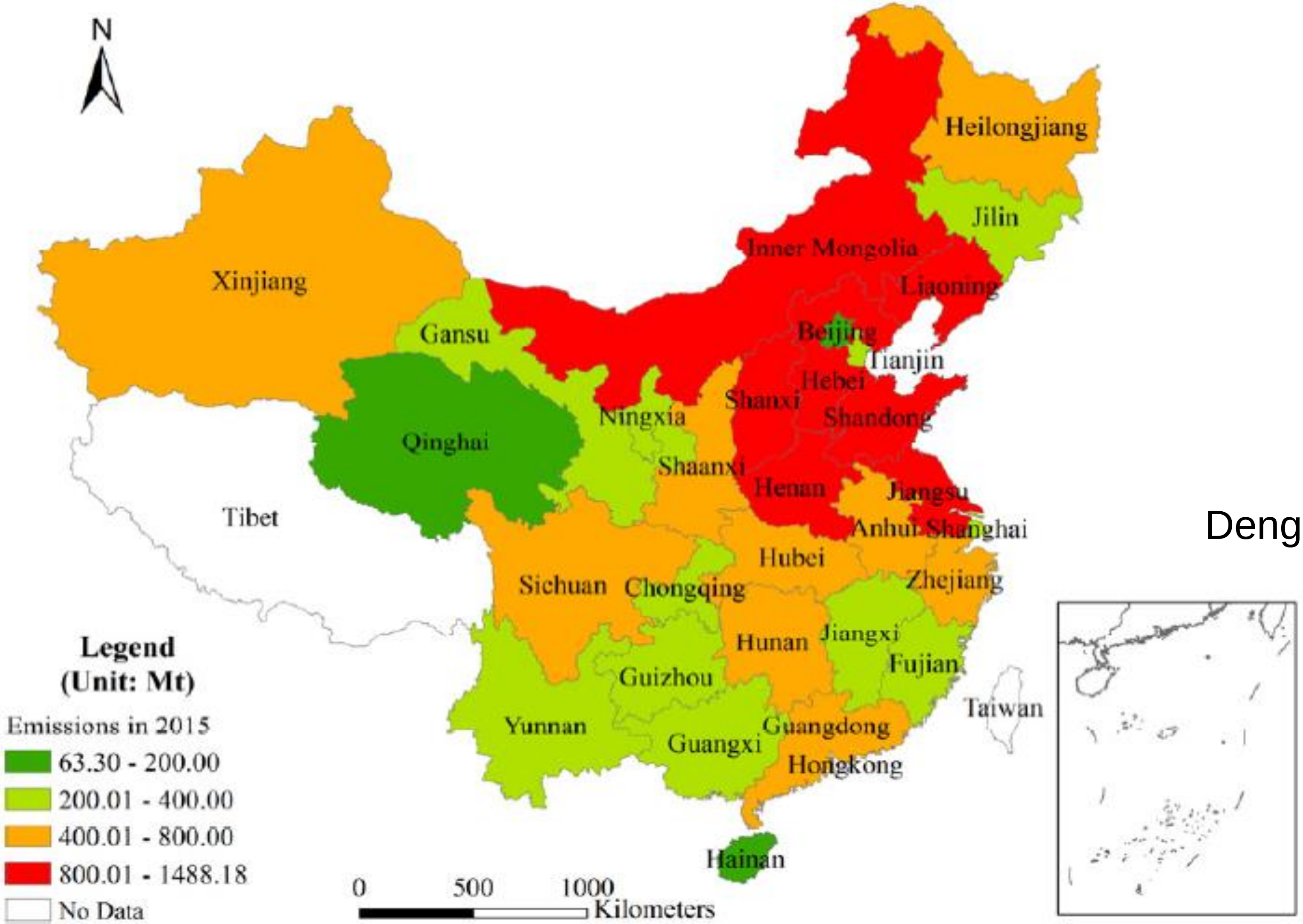
$\lambda_{kr}, x_{ir}, y_r \geq 0, \forall k \in K, i \in I_v$

## the MiCP is corrected by following two equations

- $0 \leq \lambda_{kr} \perp -\sum_{i\in I_f} \varphi 1_{ir} X_{ik} - \sum_{i\in I_v} \varphi 2_{ir} X_{ik} + \varphi 3_r Y_k - \varphi 4_r B_k - \varphi 9_r \leq 0, \forall k, r \in K$
- $0 \leq \varphi 9_r \perp \sum_{k\in K} \lambda_{kr} - 1 \leq 0, \quad \forall r \in K$

# Empirical Study

- Emission intensity distribution of CO2 in 2015;




Deng et al. (2015)

## Plant-level coal-fired power plants operating in China in 2013

- North and the Northeast regions
- investigate 33 coal-burning power plants with the nameplate capacity larger than 1M kW
- 1% emission reduction; that is, $C = 0.01 \times \sum_{k \in K} B_k = \sum_{r \in K} \Delta b_r$

## Inputs, Outputs and Prices

- Desirable output: the annual amount of electricity generated in Megawatt-hours (MWh).
  - The electricity price $P^Y(\hat{Y}) := 4.5 \times 10^7 - 123\hat{Y}$ (unit: CNY$ per $10^8$ kWh)
- Undesirable output: the annual amount in tonnes of $CO_2$.
- Fixed input: nameplate capacity
  - the price of nameplate capacity is RMB$ 250,000 per MW per year
- Variable input: the annual amount in tonnes of coal consumption
  - The price of coal is CNY$590 per tonne.

# Empirical Study

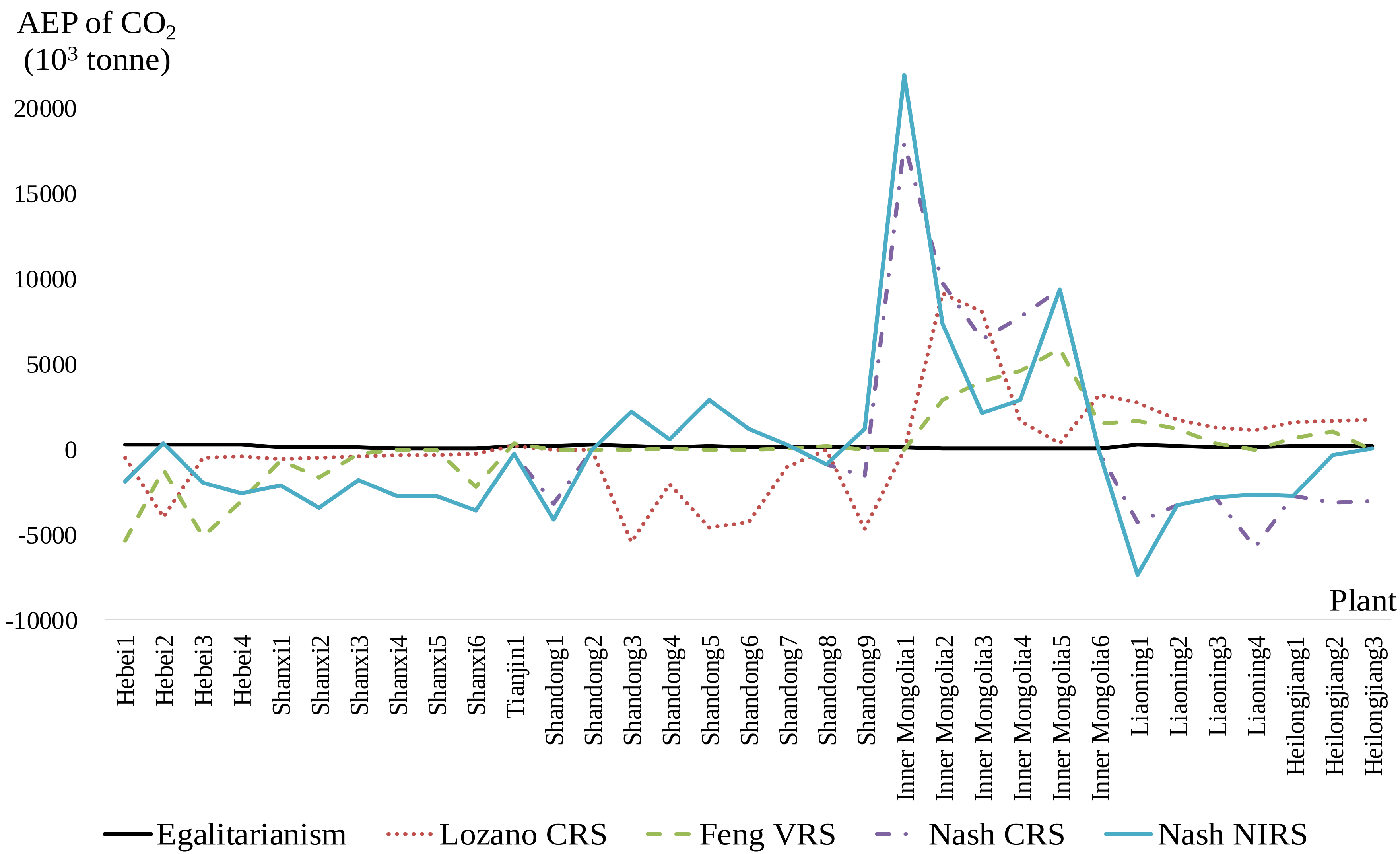
- Egalitarianism
  - based on population size
- Hebei, Shanxi, and Tianjin
  - present the negative AEP (i.e. allow to produce more emission)
- Inner Mongolia
  - positive values in five AEP models indicate urgent emission reduction.
- The inconsistent AEP occur between centralized and decentralized models.
  - The result provides the insight implying that a totally different decision could happen between different market assumptions.

| AEP | Centralized Models | | | Decentralized Models | |
|---|---|---|---|---|---|
| Plant | Egalitarianism | Lozano CRS | Feng VRS | Nash CRS | Nash NIRS |
| Hebei1 | 272.4 | -490.9 | -5337.8 | -1923.1 | -1923.1 |
| Hebei2 | 266.8 | -3955.6 | -1166.6 | 370.2 | 370.2 |
| Hebei3 | 265.0 | -479.2 | -5150.7 | -2003.1 | -2003.1 |
| Hebei4 | 250.3 | -453.1 | -3056.2 | -2596.8 | -2596.8 |
| Shanxi1 | 115.5 | -549.6 | -668.1 | -2155.3 | -2155.3 |
| Shanxi2 | 97.3 | -463.5 | -1667.0 | -3468.0 | -3468.0 |
| Shanxi3 | 95.7 | -456.9 | -256.0 | -1822.3 | -1822.3 |
| Shanxi4 | 72.0 | -342.6 | 0.0 | -2719.7 | -2719.7 |
| Shanxi5 | 71.2 | -341.4 | -58.1 | -2748.6 | -2748.6 |
| Shanxi6 | 64.1 | -305.8 | -2204.9 | -3588.8 | -3588.8 |
| Tianjin1 | 157.7 | 198.5 | 323.8 | -301.4 | -301.4 |
| Shandong1 | 203.3 | 0.0 | 0.0 | -3175.8 | -4144.8 |
| Shandong2 | 241.6 | 0.0 | 0.0 | 0.0 | 0.0 |
| Shandong3 | 165.0 | -5410.2 | 0.0 | 2208.6 | 2208.6 |
| Shandong4 | 142.6 | -2076.1 | 15.7 | 573.2 | 573.2 |
| Shandong5 | 169.0 | -4587.0 | 0.0 | 2882.5 | 2882.5 |
| Shandong6 | 150.5 | -4244.7 | 0.0 | 1172.0 | 1172.0 |
| Shandong7 | 125.4 | -1023.1 | 72.3 | 282.5 | 282.5 |
| Shandong8 | 108.3 | 0.0 | 197.2 | -865.2 | -865.2 |
| Shandong9 | 116.2 | -4675.4 | 0.0 | -1555.1 | -1216.2 |
| Inner Mongolia1 | 128.0 | 0.0 | 0.0 | 17926.6 | 21990.6 |
| Inner Mongolia2 | 65.2 | 9154.1 | 2912.3 | 9793.5 | 7372.1 |
| Inner Mongolia3 | 54.5 | 8076.8 | 3958.7 | 6462.4 | 2119.3 |
| Inner Mongolia4 | 54.5 | 1646.4 | 4578.8 | 7767.9 | 2922.8 |
| Inner Mongolia5 | 48.6 | 381.2 | 5902.9 | 9344.5 | 9344.5 |
| Inner Mongolia6 | 21.6 | 3200.8 | 1484.5 | -157.0 | -157.0 |
| Liaoning1 | 261.2 | 2754.1 | 1631.5 | -4254.9 | -7397.0 |
| Liaoning2 | 167.9 | 1767.5 | 1206.6 | -3269.1 | -3269.1 |
| Liaoning3 | 123.8 | 1307.7 | 373.4 | -2804.7 | -2804.7 |
| Liaoning4 | 105.2 | 1104.1 | 0.0 | -5719.1 | -2662.3 |
| Heilongjiang1 | 213.0 | 1585.2 | 676.6 | -2719.2 | -2719.2 |
| Heilongjiang2 | 189.0 | 1696.3 | 1008.5 | -3140.0 | -368.6 |
| Heilongjiang3 | 195.0 | 1759.5 | 0.0 | -3019.2 | 37.7 |
| # of reduction | 33 | 14 | 15 | 11 | 13 |
| Std. Dev. | 71.5 | 2997.6 | 2210.4 | 4900.5 | 4992.1 |

# Empirical Study

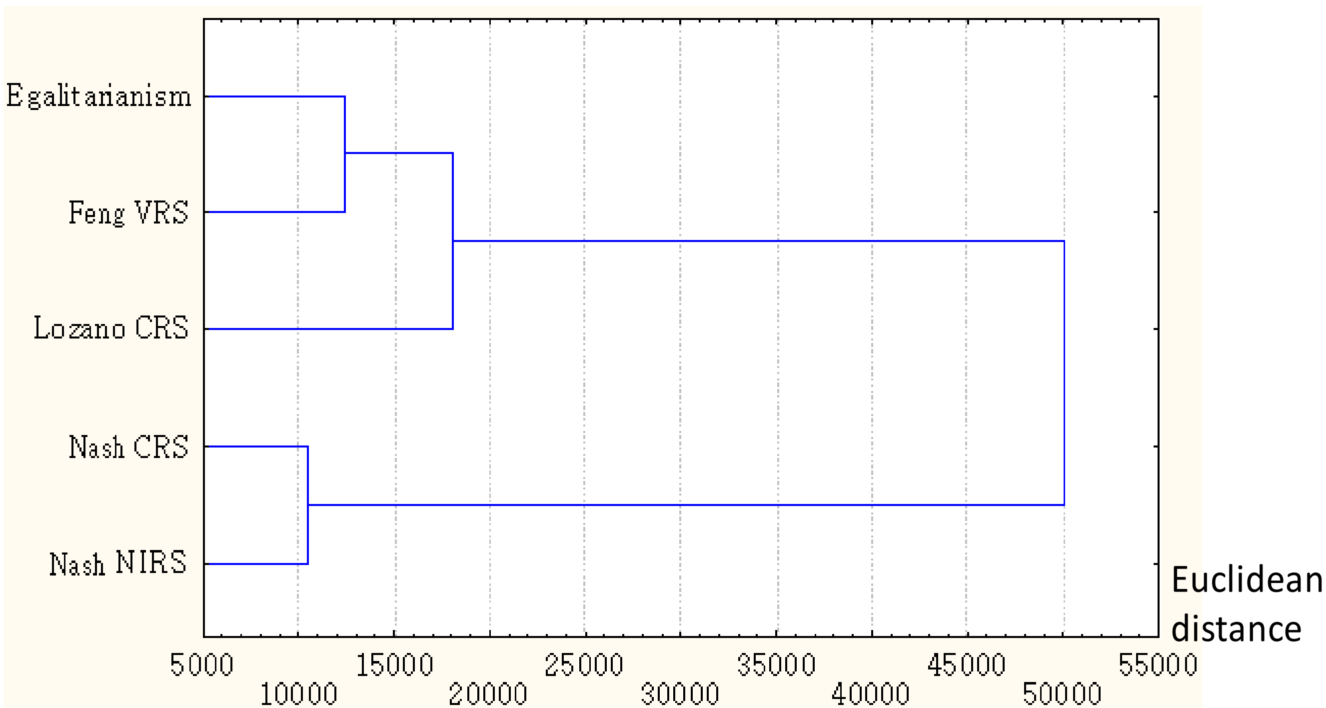
PОLab

## AEP results of the five models



# Empirical Study

- Ward's method (Clustering Analysis)
  - The results from using Egalitarianism, Lozano CRS, and Feng VRS are highly-correlated since they use centralized model without price information while Nash CRS and Nash NIRS models are similar due to decentralized model with endogenous price.



# Empirical Study

- Efficiency before and after AEP

$$\text{Min } \theta$$
$$\text{s.t. } \sum_{k\in K} \lambda_k X_{ik} \le \xi X_{ir}$$
$$\sum_{k\in K} \lambda_k Y_k \ge Y_r$$
$$\sum_{k\in K} \lambda_k B_k = \theta B_r$$
$$\sum_{k\in K} \lambda_k = \xi;$$
$$\lambda_k \ge 0, \forall k$$

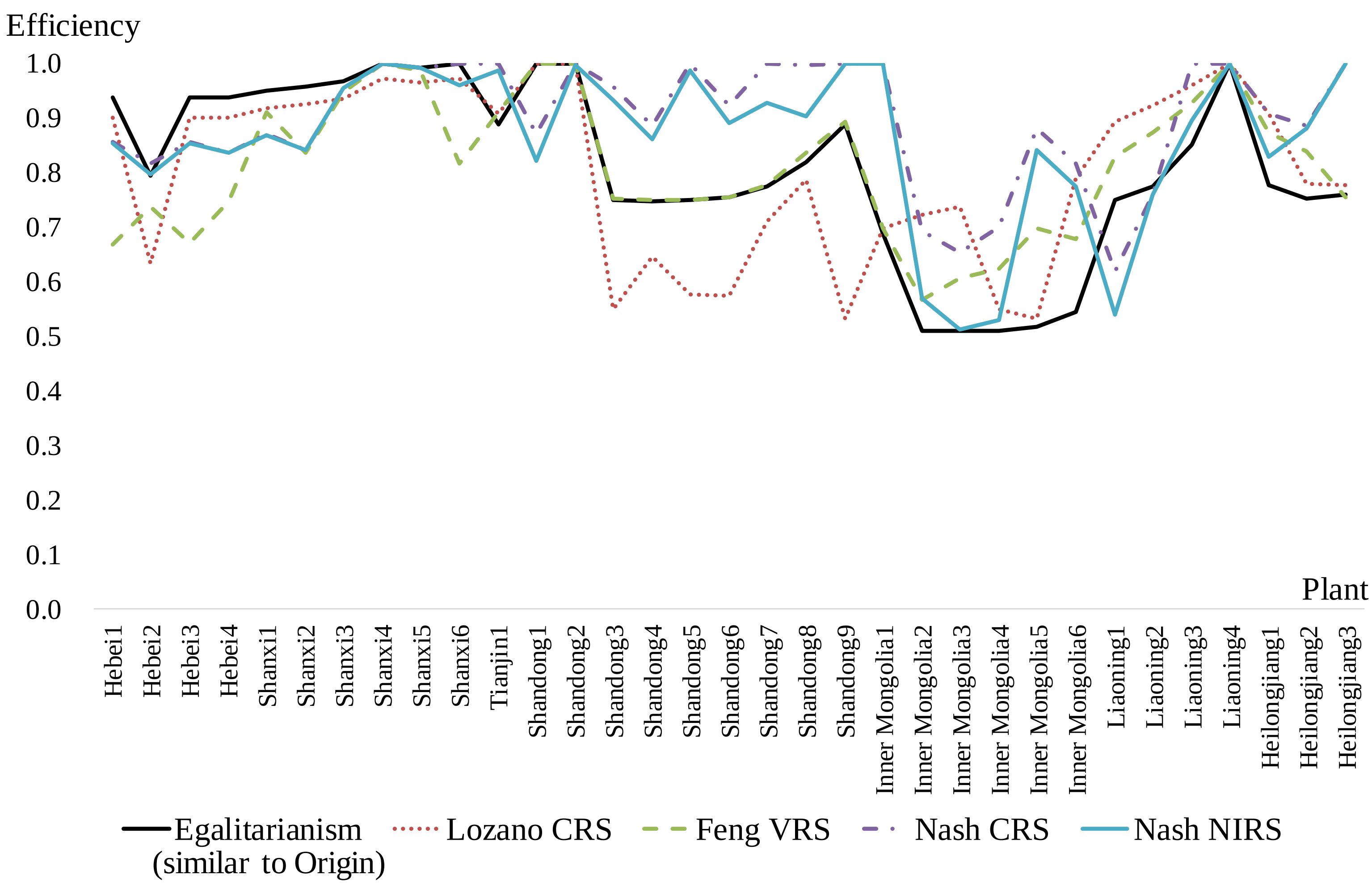
- Hebei and Shanxi present better efficiency while plants in Inner Mongolia present poor performance.

| Efficiency | Before AEP | After AEP | | | | |
|---|---|---|---|---|---|---|
| Plant | Origin | Egalitarianism | Lozano CRS | Feng VRS | Nash CRS | Nash NIRS |
| Hebei1 | 0.934 | 0.938 | 0.901 | 0.668 | 0.855 | 0.853 |
| Hebei2 | 0.793 | 0.795 | 0.632 | 0.738 | 0.818 | 0.798 |
| Hebei3 | 0.932 | 0.937 | 0.900 | 0.669 | 0.855 | 0.853 |
| Hebei4 | 0.932 | 0.937 | 0.901 | 0.748 | 0.837 | 0.836 |
| Shanxi1 | 0.956 | 0.950 | 0.918 | 0.909 | 0.870 | 0.868 |
| Shanxi2 | 0.962 | 0.956 | 0.925 | 0.836 | 0.841 | 0.841 |
| Shanxi3 | 0.970 | 0.967 | 0.936 | 0.948 | 0.955 | 0.954 |
| Shanxi4 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 |
| Shanxi5 | 0.993 | 0.993 | 0.965 | 0.987 | 0.993 | 0.991 |
| Shanxi6 | 1.000 | 1.000 | 0.973 | 0.817 | 1.000 | 0.959 |
| Tianjin1 | 0.886 | 0.888 | 0.908 | 0.911 | 1.000 | 0.988 |
| Shandong1 | 1.000 | 1.000 | 1.000 | 1.000 | 0.872 | 0.822 |
| Shandong2 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 |
| Shandong3 | 0.751 | 0.751 | 0.550 | 0.751 | 0.958 | 0.933 |
| Shandong4 | 0.748 | 0.748 | 0.647 | 0.749 | 0.886 | 0.861 |
| Shandong5 | 0.749 | 0.749 | 0.577 | 0.749 | 1.000 | 0.987 |
| Shandong6 | 0.754 | 0.754 | 0.574 | 0.754 | 0.923 | 0.890 |
| Shandong7 | 0.772 | 0.774 | 0.711 | 0.778 | 1.000 | 0.928 |
| Shandong8 | 0.819 | 0.819 | 0.786 | 0.836 | 0.997 | 0.903 |
| Shandong9 | 0.892 | 0.889 | 0.531 | 0.892 | 1.000 | 1.000 |
| Inner Mongolia1 | 0.697 | 0.688 | 0.697 | 0.697 | 1.000 | 1.000 |
| Inner Mongolia2 | 0.516 | 0.509 | 0.722 | 0.568 | 0.692 | 0.570 |
| Inner Mongolia3 | 0.517 | 0.510 | 0.738 | 0.606 | 0.654 | 0.512 |
| Inner Mongolia4 | 0.517 | 0.510 | 0.551 | 0.623 | 0.699 | 0.530 |
| Inner Mongolia5 | 0.524 | 0.517 | 0.533 | 0.697 | 0.880 | 0.841 |
| Inner Mongolia6 | 0.549 | 0.545 | 0.789 | 0.678 | 0.816 | 0.775 |
| Liaoning1 | 0.751 | 0.750 | 0.892 | 0.829 | 0.619 | 0.539 |
| Liaoning2 | 0.772 | 0.775 | 0.924 | 0.873 | 0.760 | 0.760 |
| Liaoning3 | 0.848 | 0.852 | 0.961 | 0.928 | 1.000 | 0.897 |
| Liaoning4 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Heilongjiang1 | 0.766 | 0.777 | 0.909 | 0.873 | 0.908 | 0.830 |
| Heilongjiang2 | 0.748 | 0.752 | 0.778 | 0.839 | 0.886 | 0.880 |
| Heilongjiang3 | 0.755 | 0.759 | 0.777 | 0.755 | 1.000 | 1.000 |
| # of efficient | 5 | 5 | 3 | 4 | 11 | 5 |
| Avg. | 0.812 | 0.812 | 0.805 | 0.809 | 0.896 | 0.860 |
| Std. Dev. | 0.155 | 0.157 | 0.156 | 0.122 | 0.111 | 0.139 |

# Empirical Study

POLab

- Efficiency analysis of the five AEP models



# Conclusion

## Summary

- The proposed decentralized models consider all plants compete with each other and the "invisible hand" (interpreted by Nash equilibrium) makes AEP more efficient in an imperfectly competitive power market.

- AEP is not only an optimization method for reallocation but also brings an opportunity for improving efficiency indeed.

- Knowing the AEP provides useful environmental policy guidelines
  - allowance price in emissions trading markets
  - the penalty rates for emissions

- In practice, the reallocation of emission permits is likely to meet with resistance from negatively affected plants and may also lead to an increase in monitoring costs to guarantee the reliability of the plant.

# Nash Allocation of Emission Permits

POLab

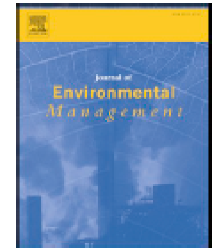
Journal of Environmental Management 241 (2019) 353–362

Contents lists available at ScienceDirect

Journal of Environmental Management

journal homepage: www.elsevier.com/locate/jenvman

ELSEVIER




Research article

## Decentralized allocation of emission permits by Nash data envelopment analysis in the coal-fired power market

Check for updates

Chia-Yen Lee

*Institute of Manufacturing Information and Systems, National Cheng Kung University, Tainan City, 701, Taiwan*

A R T I C L E  I N F O

*Keywords:*
Data envelopment analysis
Emission permit
Nash equilibrium
Decentralized allocation
Coal-fired power plant

A B S T R A C T

Allocation of emission permits (AEP) provides valuable guidelines to support environmental regulatory policies for pollutant emission, in particular, $CO_2$ as the key contributors to climate change. Most of previous studies in literature developed the centralized AEP model and focused on the coal-fired power market, one of the main sources of air pollution. However, the power market is usually imperfectly competitive and some of them are gradually deregulated, this justifies the motivation of developing a decentralized AEP model. This study proposes a decentralized AEP model which suggests Nash equilibrium as an allocatively efficient benchmark in an imperfectly competitive market with endogenous price. The proposed model is formulated by data envelopment analysis (DEA) and transformed into the mixed complementarity problem (MiCP) for identifying the Nash equilibrium. A study of coal-fired power plants operating in China in 2013 is conducted and the results show that the decentralized model complements the centralized model; in particular, with considering the price and market structure, the proposed decentralized model described in this study investigates the potential for efficiency improvement after AEP among the coal-fired plants.

# Conclusion

## Nash Equilibrium Identified in DEA

- For imperfectly competitive market; in particular, energy market
- Mixed Complementarity Problem (MiCP)
- Existence and uniqueness

## Proposed Models

- Case1: Nash-Profit Efficiency Measuring Market Structures
- Case2: Mixed-Strategy Nash Equilibrium
- Case3: Nash Shadow Price Estimation
- Case4: Allocation of Emission Permits

## Remarks

- AEP and MAC are not only an optimization method for reallocation but also bring an opportunity for improving efficiency indeed.
- Knowing the AEP provides useful environmental policy guidelines
  - allowance price in emissions trading markets
  - the penalty rates for emissions

# Thanks for your attention

Contact Information:
name: 李家岩 (Chia-Yen Lee)
phone: 886-6-2757575 Ext. 34223
email: cylee@mail.ncku.edu.tw
web: https://polab.imis.ncku.edu.tw/